The Diurnal Efficiency of a Flat Plate Solar Collector with Thick Absorber

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Abstract: *The efficiency of a flat plate solar collector with thick absorber is estimated.*

The heat energy equation is solved using the Fourier Series Expansion Technique. The Fourier's law and heat balance equations are also considered. The absorber is made of different materials: Copper, Aluminum, Silicon Carbide and Mica respectively. Different physical and cooling conditions are also highlighted. The temperature of the working fluid (water) with different rate flows is computed along the local day time. A published formula that predicts the incident global solar irradiance is considered in the computations.

Factors affecting the efficiency are revealed.

Keywords: *Solar collector; Heat transfer; Heat diffusion; Fourier series expansion technique; Performance; Efficiency.*

1. Introduction

The flat plate collectors are still among the widely spread designs used to capture the incident solar insolation. This absorbed solar energy is transferred to the working fluid sweeping the rear surface of its absorber, the front surface of which is subjected to incident solar irradiance.

The efficiency of such a device, is defined as the ratio between the absorbed solar energy to the energy gained by the working fluid that flows through a container (reservoir), the absorber represents its upper ceiling. Such efficiency has aroused the interest of many investigators [1-6].

The main part of the flat solar collector is its absorber plate. There are many factors that affect the efficiency of the considered device. Among these factors, physical and geometrical parameters of the absorber plate, its optical absorptivity, the different modes of heat loss, the kind of the working fluid, and its rate of mass flow.

Information on the received diurnal global solar irradiance $q_0(t)$, W/m^2 is required. A previous trial is made by [7] to study the effciency of a flat plate solar collector with a thin metallic absorber. The thermal problem based on solving a heat balance equation. The present trial is oriented to study the same collector with thick absorber. Thus, the mathematical treatment is based on solving the heat diffusion equation.

2. Heating Problem

2-1- Determination of the temperature of the rear surface of the thick flat plate collector

To study the performance, a simple model for the flat plate collector is considered as shown in fig. (1)

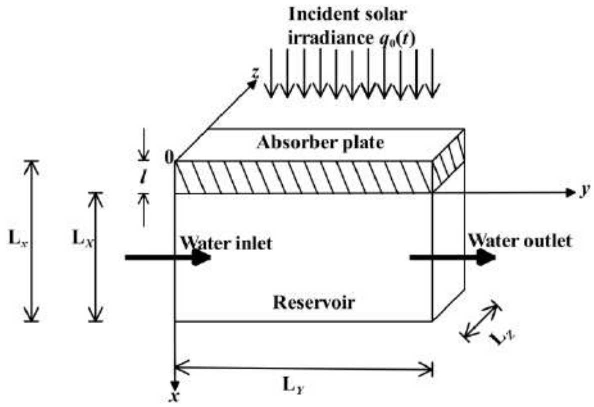

Fig. (1): A flat plate solar collector

In setting up the problem, it is assumed that solar radiations of irradiance $q_0(t), W/m^2$ is incident on the upper surface of the flat absorber of thickness l, m. This function$q_0(t)$ is considered in the form[8]:

$$q_0(t) = 4q_{max}\left(\frac{t}{t_d}\right)\left(1-\frac{t}{t_d}\right) \quad (1)$$

Where,q_{max}, W/m^2 is the maximumvalue of the global solar irradiance along the local day time, t is the local day time, and t_d is the length of the solar day[9]given as:

$t_d = \frac{24}{180}\cos^{-1}(-\tan\delta\tan L)$, where δ is the solar declination, L is the latitude.

Such solar insolation is partly reflected and partly absorbed at the front surface of the absorber. A temperature gradient will be generated across the thick absorber. Let the absorbed part of the incident solar irradiance is $Aq_0(t)$

where $A = (1-R)$ stands for the optical absorption coefficient at the surface of the absorber, R is the reflectivity. Selective coating may be recommended to increase the absorbed part. The heat energy equation for the finite absorber can be written as follows:

$$\frac{\partial q(x,t)}{\partial t} = \alpha \frac{\partial^2 q(x,t)}{\partial x^2} \quad 0 \le x \le l,\ t > 0 \tag{2}$$

Where, $\alpha = \frac{\lambda}{\rho c_p}$, m^2/s, is the thermal diffusivity of the absorber material in terms of the thermal conductivity λ, W/mK, and the heat capacity per unit volume ρc_p.

Equation (2) is subjected to the following initial condition:

$$At \quad t = 0 \quad q_0(0) = 0 \tag{3}$$

Together with the following boundary condition (Fourier's law):

$$At \quad x = 0 \quad Aq_0(t) = -\lambda \frac{\partial \theta(x,t)}{\partial x}\Big|_{x=0} \tag{4}$$

Moreover, the following heat balance equation has to be considered:

$$\int_0^t Aq_0(t)dt = \int_0^l \rho c_p \theta(x,t)dx + \int_0^t h\theta(l,t)dt \tag{5}$$

Where, $\theta(x,t) = (T(x,t) - T_0)$, is the excess temperature relative to the ambient temperature T_0 and $h, W/m^2K$ is the heat transfer coefficient for convection at the rear surface of the absorber.Heat losses due to radiation or convection at the front surface are neglected.

To solve equation (2), one has to transform it into a nonhomogeneous equation subjected to homogenous boundary conditions encountered on the spatial variables x. This can be realized by suggesting that the required solution can be given in the form[10]:

$$q(x,t) = V(x,t) + W(x,t) \tag{6}$$

Substituting eq. (6) into eq. (2), one gets:

$$\frac{\partial^2 W}{\partial x^2} - \frac{1}{\alpha}\frac{\partial W}{\partial t} = -\left(\frac{\partial^2 V}{\partial x^2} - \frac{1}{\alpha}\frac{\partial V}{\partial t}\right) \tag{7}$$

For such a case, the boundary conditions will be given in the form:

$$W(0,t) = Aq_0(t) - V(0,t) \tag{8}$$

$$W(l,t) = q(l,t) - V(l,t) \tag{9}$$

The function $V(x,t)$ has to be chosen so that the boundary conditions (8) and (9) become homogenous.

This can be realized by assuming $V(x,t)$ in the form:

$$V(x,t) = Aq_0(t) + \frac{x}{l}\{q(l,t) - Aq_0(t)\} \tag{10}$$

This gives:

$$W(x,0) = 0 \tag{11}$$

$$W(0,t) = 0 \tag{12}$$

$$W(l,t) = 0 \tag{13}$$

Substituting equation (10) into equation (7), one gets the equation for $W(x,t)$ in the form:

$$\frac{\partial^2 W}{\partial x^2} - \frac{1}{\alpha}\frac{\partial W}{\partial t} = F(x,t) \tag{14}$$

Where,

$$F(x,t) = \frac{1}{\alpha}\left[A\dot{q}_0(t) + \frac{x}{l}\{\dot{q}(l,t) - A\dot{q}_0(t)\}\right] \tag{15}$$

To solve equation (14), we shall choose the Fourier series expansions technique[10].

According to which, let:

$$W(x,t) = \sum_{n=1}^{\infty} W_n(t)\sin\frac{n\pi x}{l} \tag{16}$$

$$F(x,t) = \sum_{n=1}^{\infty} F_n(t)\sin\frac{n\pi x}{l} \tag{17}$$

Inserting equations (16), (17) into equation (14), one gets:

$$\sum_{n=1}^{\infty}\left[\left(\frac{n\pi}{l}\right)^2 W_n(t) + \frac{1}{\alpha}\dot{W}_n(t) + F_n(t)\right]\sin\frac{n\pi x}{l} = 0 \tag{18}$$

Since $\sin\frac{n\pi x}{l} \neq 0$, one gets

$$\dot{W}_n(t) + \alpha\left(\frac{n\pi}{l}\right)^2 W_n(t) = -\alpha F_n(t) \tag{19}$$

Equation (19) has an integrating factor $e^{\int \alpha\left(\frac{n\pi}{l}\right)^2 dt}$, the solution will be given in the form [11]:

$$W_n(t) = e^{-at}\left[\int_0^t -\alpha F_n(t)e^{at}dt\right] \tag{20}$$

Where, $a = \alpha\left(\frac{n\pi}{l}\right)^2$

From equations (15) and equation (17), one gets:

$$\frac{1}{\alpha}\left[A\dot{q}_0(t) + \frac{x}{l}\{\dot{q}(l,t) - A\dot{q}_0(t)\}\right] = \sum_{n=1}^{\infty} F_n(t)\sin\frac{n\pi x}{l} \tag{21}$$

To find the expression for $F_n(t)$ let us apply the properties of the orthonormal functions [10],

Multiplying both sides by $\sin\frac{m\pi x}{l}$ and performing the required integrations, one finally gets:

$$F_n(t) = \left(\frac{2}{n\pi\alpha}\right)\{\dot{q}(l,t) + A\dot{q}_0(t)\} \tag{22}$$

Inserting equation (22) into equation (20), and performing the required integrations, one gets the solution $W_n(t)$ in the form:

$$W_n(t) = \left(\frac{2}{n\pi}\right)\left[-q(l,t) - Aq_0(t) + ae^{-at}\int_0^t e^{at}q(l,t)dt + aAe^{-at}\int_0^t e^{at}q_0(t)dt\right] \tag{23}$$

Substituting from (10), (16) into (6), one gets:

$$q(x,t) = Aq_0(t) + \frac{x}{l}\{q(l,t) - Aq_0(t)\} + \sum_{n=1}^{\infty} W_n(t) \sin\frac{n\pi x}{l} \quad (24)$$

The temperature profile within the absorber can be obtained using the Fourier law in an integral form as:

$$\int_{\theta(0,t)}^{\theta(x,t)} d\theta = \int_0^x -\frac{q(x,t)}{\lambda} dx \quad (25)$$

This gives:

$$\theta(x,t) - \theta(0,t) = -\frac{1}{\lambda}\int_0^x q(x,t)dx \quad (26)$$

Substituting from (24) into (26), one gets:

$$\theta(x,t) - \theta(0,t) = -\frac{1}{\lambda}\left[Aq_0(t)\int_0^x dx + \frac{1}{l}\{q(l,t) - Aq_0(t)\}\int_0^x xdx + \sum_{n=1}^{\infty} W_n(t)\int_0^x \sin\frac{n\pi x}{l} dx\right] \quad (27)$$

Then;

$$\theta(x,t) = \theta(0,t) \; -\frac{1}{\lambda}\left[Aq_0(t)x + \frac{x^2}{2l}\left(q(l,t) - Aq_0(t)\right) + \sum_{n=1}^{\infty} W_n(t)\left(\frac{-l}{n\pi}\right)\left(\cos\frac{n\pi x}{l} - 1\right)\right] \quad (28)$$

<u>Determination of $\theta(0,t)$:</u>

Substituting for $\theta(x,t)$ from eq. (28) into the heat balance equation eq. (5), one gets:

$$\int_0^t Aq_0(t)dt = \int_0^l \rho c_p\theta(0,t)dx - \frac{\rho c_p}{\lambda}\int_0^l \left\{Aq_0(t)x + \left(\frac{x^2}{2l}\right)\left(q(l,t) - Aq_0(t)\right) + \sum_{n=1}^{\infty} W_n(t)\left(\frac{-l}{n\pi}\right)\left(\cos\frac{n\pi x}{l} - 1\right)\right\} dx + \int_0^t h\theta(l,t)dt \quad (29)$$

Then:

$$\int_0^t Aq_0(t)dt = l\rho c_p\theta(0,t) - \frac{\rho c_p}{\lambda}\left\{\frac{Aq_0(t)l^2}{2} + \frac{l^2}{6}\left(q(l,t) - Aq_0(t)\right) + \sum_{n=1}^{\infty} W_n(t)\left(\frac{l^2}{n\pi}\right)\right\} + \int_0^t h\theta(l,t)dt \quad (30)$$

Equation (30) makes it possible to determine $\theta(0,t)$ in the form:

$$\theta(0,t) = \frac{1}{l\rho c_p}\int_0^t Aq_0(t)dt + \frac{1}{l\lambda}\left\{\frac{Aq_0(t)l^2}{2} + \frac{l^2}{6}\left(q(l,t) - Aq_0(t)\right) + \sum_{n=1}^{\infty} W_n(t)\left(\frac{l^2}{n\pi}\right)\right\} - \int_0^t \frac{h}{l\rho c_p}\theta(l,t)dt \quad (31)$$

Substituting eq. (31) in equation (28), one finally gets the following expression for $\theta(x,t)$:

$$\theta(x,t) = \frac{1}{l\rho c_p}\int_0^t Aq_0(t)dt + \frac{1}{l\lambda}\left\{\frac{Aq_0(t)l^2}{2} + \frac{l^2}{6}\left(q(l,t) - Aq_0(t)\right) + \sum_{n=1}^{\infty} W_n(t)\left(\frac{l^2}{n\pi}\right)\right\} - \int_0^t \frac{h}{l\rho c_p}\theta(l,t)dt - \frac{1}{\lambda}\left[Aq_0(t)x + \frac{x^2}{2l}\left(q(l,t) - Aq_0(t)\right) + \sum_{n=1}^{\infty} W_n(t)\left(\frac{-l}{n\pi}\right)\left(\cos\frac{n\pi x}{l} - 1\right)\right] \quad (32)$$

Substituting for $x = l$ in equation (32), and taking into consideration that$q(l,t) = h\theta(l,t)$one gets an equation for $\theta(l,t)$ in the form:

$$\therefore \quad \theta(l,t) = \frac{A}{l\rho c_p}\int_0^t q_0(t)dt + \frac{lA}{2\lambda}q_0(t) + \frac{l}{6\lambda}q(l,t) - \frac{lA}{6\lambda}q_0(t) + \frac{l}{\lambda}\sum_{n=1}^{\infty}\frac{W_n(t)}{n\pi} - \frac{h}{l\rho c_p}\int_0^t \theta(l,t)dt - \frac{lA}{\lambda}q_0(t) - \frac{l}{2\lambda}q(l,t) + \frac{lA}{2\lambda}q_0(t) - \frac{2l}{\lambda}\sum_{n=1}^{\infty}\left(\frac{W_n(t)}{n\pi}\right), \quad for\ n = odd \quad (33)$$

$$\theta(l,t) = \frac{A}{l\rho c_p}\int_0^t q_0(t)dt - \frac{lA}{6\lambda}q_0(t) - \frac{l}{\lambda}\sum_{n=1}^{\infty}\frac{W_n(t)}{n\pi} - \frac{h}{l\rho c_p}\int_0^t \theta(l,t)dt - \frac{l}{3\lambda}q(l,t) \quad (34)$$

Rearranging eq. (34), one gets:

$$\theta(l,t) + \frac{lh}{3\lambda}\theta(l,t) + \frac{1}{l\rho c_p}\int_0^t h\theta(l,t)dt = \frac{A}{l\rho c_p}\int_0^t q_0(t)dt - \frac{lA}{6\lambda}q_0(t) - \frac{l}{\lambda}\sum_{n=1}^{\infty}\frac{1}{n\pi}W_n(t) \quad (35)$$

Substituting for $W_n(t)$from eq. (23) into eq. (35), and considering that $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$ [12], one gets:

$$\theta(l,t) + \frac{l}{3\lambda}h\theta(l,t) + \frac{1}{l\rho c_p}\int_0^t h\theta(l,t)dt = \frac{A}{l\rho c_p}\int_0^t q_0(t)dt - \frac{lA}{6\lambda}q_0(t) + \frac{l}{3\lambda}h\theta(l,t) + \frac{l}{3\lambda}Aq_0(t) - I_1 - I_2 \quad (36)$$

Where,

$$I_1 = \frac{l}{\lambda}\sum_{n=1}^{\infty}\frac{2}{(n\pi)^2}ae^{-at}\int_0^t e^{at}q_0(t)dt \quad (37)$$

$$I_2 = \frac{l}{\lambda}\sum_{n=1}^{\infty}\frac{2}{(n\pi)^2}ae^{-at}\int_0^t e^{at}q(l,t)dt \quad (38)$$

Substituting for $q_0(t)$ from eq. (1) into eq. (39) and performing the required integrations [13], one gets:

$$I_1 = \frac{l}{\lambda}\sum_{n=1}^{\infty}\frac{2}{(n\pi)^2}ae^{-at}\int_0^t e^{at}(4q_{max})\left(\frac{t}{t_d}\right)\left(1 - \frac{t}{t_d}\right)dt = \left(\frac{8Aq_{max}}{l\rho c_p t_d}\right)\sum_{n=1}^{\infty}\left\{\left(\frac{t}{a} - \frac{1}{a^2} + \frac{1}{a^2}e^{-at}\right) - \frac{1}{t_d}\left(\frac{t^2}{a} - \frac{2t}{a^2} + \frac{2}{a^3} - \frac{2}{a^3}e^{-at}\right)\right\} \quad (39)$$

Evaluating the value of the expression for I_1, one finds that it is of order$O(-6)$.

Moreover, since, $q(l,t) < q_0(t)$, thus,$I_2 < I_1$

Hence, one can neglect both terms I_2andI_1 with respect to the other terms in eq. (36).

Finally, one gets the equation for $\theta(l,t)$ in the form:

$$\theta(l,t) + \frac{h}{l\rho c_p}\int_0^t \theta(l,t)dt = \frac{A}{l\rho c_p}\int_0^t q_0(t)dt + \frac{lA}{6\lambda}q_0(t) \quad (40)$$

To get an expression for $\theta(l,t)$ let us differentiate both sides of eq. (40), this gives:

$$\frac{d}{dt}\theta(l,t) + \frac{h}{l\rho c_p}\theta(l,t) = \frac{A}{l\rho c_p}q_0(t) + \frac{lA}{6\lambda}\dot{q}_0(t) \qquad (41)$$

Eq. (41) has an integrating factor $e^{\int \frac{h}{l\rho c_p}dt} = e^{bt}$

Where, $b = \frac{h}{l\rho c_p}$

The solution for $\theta(l,t)$is given in the form:

$$\theta(l,t) = e^{-bt}\left[\frac{A}{l\rho c_p}\int_0^t e^{bt}q_0(t)dt + \frac{lA}{6\lambda}\int_0^t e^{bt}\dot{q_0}(t)dt\right] \qquad (42)$$

Performing the required integrations[13]in equation (42),

One finally gets the expression for the temperature of the rear surface of the absorber in the form:

$$\theta(l,t) = \frac{4Aq_{max}}{l\rho c_p t_d}\left\{\left(\frac{t}{b} - \frac{1}{b^2} + \frac{1}{b^2}e^{-bt}\right) - \frac{1}{t_d}\left(\frac{t^2}{b} - \frac{2t}{b^2} + \frac{2}{b^3} - \frac{2}{b^3}e^{-bt}\right)\right\} + \frac{2Aq_{max}l}{3\lambda t_d}\left\{\frac{1}{b}(1 - e^{-bt}) - \frac{2}{t_d}\left(\frac{t}{b} - \frac{1}{b^2} + \frac{1}{b^2}e^{-bt}\right)\right\} \qquad (43)$$

2-2- Determination of the working fluid temperature:

Let the thick absorber of thickness *"l"*, (m) represents the upper ceiling for a reservoir of dimensions $(L_X = L_x - l), L_Y$ andL_Z , (m). The upper surface of the absorber of area $S_x = L_Y L_Z$, (m^2) is subjected to the incident solar radiation $q_0(t), W/m^2$.

The x-axis is taken vertically downwards. It coincides with the direction of the incident radiation. The volume of the reservoir is $V_{res} = L_X L_Y L_Z, m^3$.

The sides of the reservoir are assumed to be thermally insulated. The working fluid enters the reservoir from the face $dS_y = L_Z L_X$, (m^2) and emerges from the opposite sides. For simplicity, let $L_Y = L_Z = 1m$.

The fluid flows along the y-direction with velocity v_y, (m/s), and volumetric rate $G_y = L_Z L_X v_y$, (m^3/s).

Let θ_w represents the average temperature of the working fluid within an interval of time Δt.

The value of which is given as:

$$\theta_w(t)average = \frac{\int_0^t \theta_w(t)\,dt}{\int_0^t dt} \qquad (44)$$

Thus, the heat balance equation concerning the working fluid within an interval of time Δt is written in the form:

$$L_Z L_Y \int_0^t h\theta(l,t)dt = V_{res}\rho_w c_{p_w}\theta_w(t) + \rho_w c_{p_w}\theta_w(t)\int_0^t G_y dt \qquad (45)$$

If the volumetric rate of working fluid is considered to be constant, i.e. $G_y = const$. One gets for the average temperature of the working fluid the expression:

$$\theta_w(t) = \frac{L_Z L_Y \int_0^t h\theta(l,t)dt}{L_Z L_X L_Y \rho_w c_{p_w} + \rho_w c_{p_w} L_Z L_X v_y t} \qquad (46)$$

Where:

$$\int_0^t h\theta(l,t)dt = \frac{2Ahq_{max}}{t_d}\left[\frac{2}{l\rho c_p}\left\{\left(\frac{t^2}{2b} - \frac{t}{b^2} + \frac{1}{b^3}(1 - e^{-bt})\right) - \frac{1}{t_d}\left(\frac{t^3}{3b} - \frac{t^2}{b^2} + \frac{2t}{b^3} - \frac{2}{b^4}(1 - e^{-bt})\right)\right\} + \frac{l}{3\lambda}\left\{\frac{1}{b}\left(t - \frac{1}{b}(1 - e^{-bt})\right) - \frac{2}{t_d}\left(\frac{t^2}{2b} - \frac{t}{b^2} + \frac{1}{b^3}(1 - e^{-bt})\right)\right\}\right] \qquad (47)$$

2-3- Determination of the effciency η:

The efficiency of the flat plate collector within a certain interval of time $\Delta t = \int_0^t dt, (s)$. defined through the equation:

$$\eta = \frac{\text{The heat energy gained by the fluid within the interval "}\Delta t\text{"}}{\text{The incident solar energy receivedwithin the same interval "}\Delta t\text{"}} = \frac{\int_0^t h\theta(l,t)dt}{\int_0^t q_0(t)dt} \qquad (48)$$

Substituting the distribution $q_0(t)$ from eq. (1) into eq. (48) one gets:

$$\eta = \frac{\int_0^t h\theta(l,t)dt}{4q_{max}t_d\left[\frac{1}{2}\left(\frac{t}{t_d}\right)^2 - \frac{1}{3}\left(\frac{t}{t_d}\right)^3\right]} \qquad (49)$$

3. The Computations

A computer program (MATLAB R2017b) is used in the following computations.

The incident solar irradiance $q_0(t)$received per unit area in Makkah (1983) [8] is considered for computations with parameters:$q_{max} = 938\ W/m^2$, $t_d = 12\ hr$, and is predicted using equation (1) with fitting 8% [8].

The area of the absorber surface is1 m^2.

Shifted time is considered according to which the sunrise time t_r is taken as zero.

The absorption coefficient = 1 .

For water as the working fluid: $\rho_w = 1000\ kg/m^3$, $c_{p_w} = 4.1818 \times 10^3 J/kg \cdot K$.

The depth of the reservoir is 0.1 m.

- The temperature of the working fluid (water)$\theta_w(t)$ is computed according to equation (46).
- The effciency "η" is computed according to equation (49).

2-4- The effect of the absorber material:

Considering the parameters $h = 300\ W/m^2K$, thickness $l = 0.01m$ and $v_y = 5 \times 10^{-6}\ m/s$, one can get $\theta_w(t)$ for the different considered materials. These are Copper (Cu), Aluminum (Al), Silicon carbide (SiC), and Mica. The physical parameters of these materials are given in table (1).

Table (1): The physical parameters of the considered absorbermaterials[14 - 15]

Element	$\rho,\ kg/m^3$	$c_p,\ J/kg.K$	$\lambda,\ W/mK$
Copper (Cu)	8954	383.1	385
Aluminum (Al)	2710	910	205
Silicon Carbide (SiC)	3210	600	125
Mica	2883	880	2.3

The obtained results are given in tables (2).

Table (2): The variation of the temperature of the working fluid $\theta_w(t)$ with local day time.

Shifted time t_s, hr	$\theta_w(t), K$			
	Cu	Al	SiC	Mica
1	1.1744	1.1950	1.2068	1.2326
2	4.4852	4.5231	4.5445	4.5911
3	9.3997	9.4505	9.4790	9.5410
4	15.4184	15.4778	15.5113	15.5836
5	22.0748	22.1390	22.1751	22.2529
6	28.9329	28.9981	29.0346	29.1135
7	35.5843	35.6469	35.6820	35.7575
8	41.6466	41.7031	41.7348	41.8028
9	46.7609	46.8081	46.8344	46.8910
10	50.5901	50.6249	50.6443	50.6856
11	52.8177	52.8370	52.8477	52.8703
12	53.1456	53.1467	53.1471	53.1475

2-5- The effect of the absorber's thickness l:

The temperature of the working fluid $\theta_w(t)$ and the efficiency η are computed for a copper with different thicknesses($l = 0.5\ , 1\ , 2\ and\ 5\ cm$) with parameters $h = 300\ W/m^2K$ and $v_y = 5 \times 10^{-6}\ m/s$.

The obtained results are given in tables (3,4).

Table (3): The variation of the temperature of the working fluid $\theta_w(t)$ with local day time.

Shifted time t_s, hr	$\theta_w(t), K$			
	$l = 0.5\ cm$	$l = 1\ cm$	$l = 2\ cm$	$l = 5\ cm$
1	1.2112	1.1744	1.1045	0.9234
2	4.5526	4.4852	4.3535	3.9825
3	9.4898	9.3997	9.2218	8.7075
4	15.5239	15.4184	15.2090	14.5957
5	22.1887	22.0748	21.8482	21.1780
6	29.0484	28.9329	28.7023	28.0154
7	35.6952	35.5843	35.3625	34.6971
8	41.7467	41.6466	41.4458	40.8380
9	46.8444	46.7609	46.5925	46.0772
10	50.6516	50.5901	50.4653	50.0756
11	52.8518	52.8177	52.7469	52.5144
12	53.1473	53.1456	53.1390	53.0938

Table (4): The variation of the efficiency η with local day time.

Shifted time	η, %

t_s, hr	$l = 0.5\ cm$	$l = 1\ cm$	$l = 2\ cm$	$l = 5\ cm$
1	96.9639	94.0214	88.4202	73.9247
2	98.5216	97.0643	94.2135	86.1852
3	99.0511	98.1103	96.2533	90.8851
4	99.3212	98.6460	97.3069	93.3830
5	99.4877	98.9771	97.9609	94.9562
6	99.6032	99.2070	98.4164	96.0611
7	99.6906	99.3810	98.7615	96.9029
8	99.7615	99.5223	99.0423	97.5901
9	99.8230	99.6451	99.2863	98.1883
10	99.8802	99.7591	99.5129	98.7445
11	99.9373	99.8729	99.7391	99.2994
12	99.9990	99.9958	99.9834	99.8984

2-6- The effect of the heat transfer coefficient for convection h:

Considering the parameters $l = 0.01\ m$ and $v_y = 5 \times 10^{-6}\ m/s$ for Copper as an example, one can getthe efficiency (η) for different values of $h(50\ , 100\ , 200\ and\ 300\ W/m^2K)$.

The obtained results are given in table (5).

Table (5): The variation of the efficiency $\eta$with local day time.

Shifted time t_s, hr	η, %			
	$h = 50$	$h = 100$	$h = 200$	$h = 300$
1	69.6248	83.1434	91.1637	94.0214
2	83.5643	91.4193	95.6234	97.0643
3	89.0831	94.4102	97.1746	98.1103
4	92.0475	95.9690	97.9724	98.6460
5	93.9242	96.9412	98.4665	98.9771
6	95.2464	97.6201	98.8101	99.2070
7	96.2557	98.1355	99.0703	99.3810
8	97.0805	98.5551	99.2818	99.5223
9	97.7990	98.9200	99.4656	99.6451
10	98.4673	99.2591	99.6363	99.7591
11	99.1340	99.5974	99.8066	99.8729
12	99.8535	99.9628	99.9906	99.9958

2-7- The effect of the flow rate of the working fluidv_y:

Considering the parameters $l = 0.01\ m$ and $h = 300\ W/m^2K$ for Copper as an example, one can get$\theta_w(t)$for different values of the rate flow $(v_y)(0.1 \times 10^{-6}\ , 1 \times 10^{-6}\ ,\ 5 \times 10^{-6}\ , 10 \times 10^{-6}\ \ and\ \ 20 \times 10^{-6}\ m/s)$.

The obtained results are given in table (6) and illustrated graphically in figure (2):

Table (6): The variation of the temperature of the working fluid$\theta_w(t)$ with local day time.

Shifted time t_s, hr	θ_w, K				
	$v_y = 0.1 \times 10^{-6}$ m/s	$v_y = 1 \times 10^{-6}$ m/s	$v_y = 5 \times 10^{-6}$ m/s	$v_y = 10 \times 10^{-6}$ m/s	$v_y = 20 \times 10^{-6}$ m/s
1	1.1952	1.1913	1.1744	1.1540	1.1153
2	4.6434	4.6135	4.4852	4.3346	4.0618
3	9.8966	9.8014	9.3997	8.9416	8.1474
4	16.5047	16.2938	15.4184	14.4480	12.8327
5	24.0183	23.6361	22.0748	20.3911	17.6923
6	31.9885	31.3798	28.9329	26.3632	22.3866
7	39.9673	39.0831	35.5843	32.0032	26.6409

8	47.5069	46.3100	41.6466	36.9905	30.2308
9	54.1607	52.6309	46.7609	41.0394	32.9710
10	59.4822	57.6220	50.5901	43.8944	34.7072
11	63.0260	60.8653	52.8177	45.3264	35.3100
12	64.3471	61.9489	53.1456	45.1292	34.6701

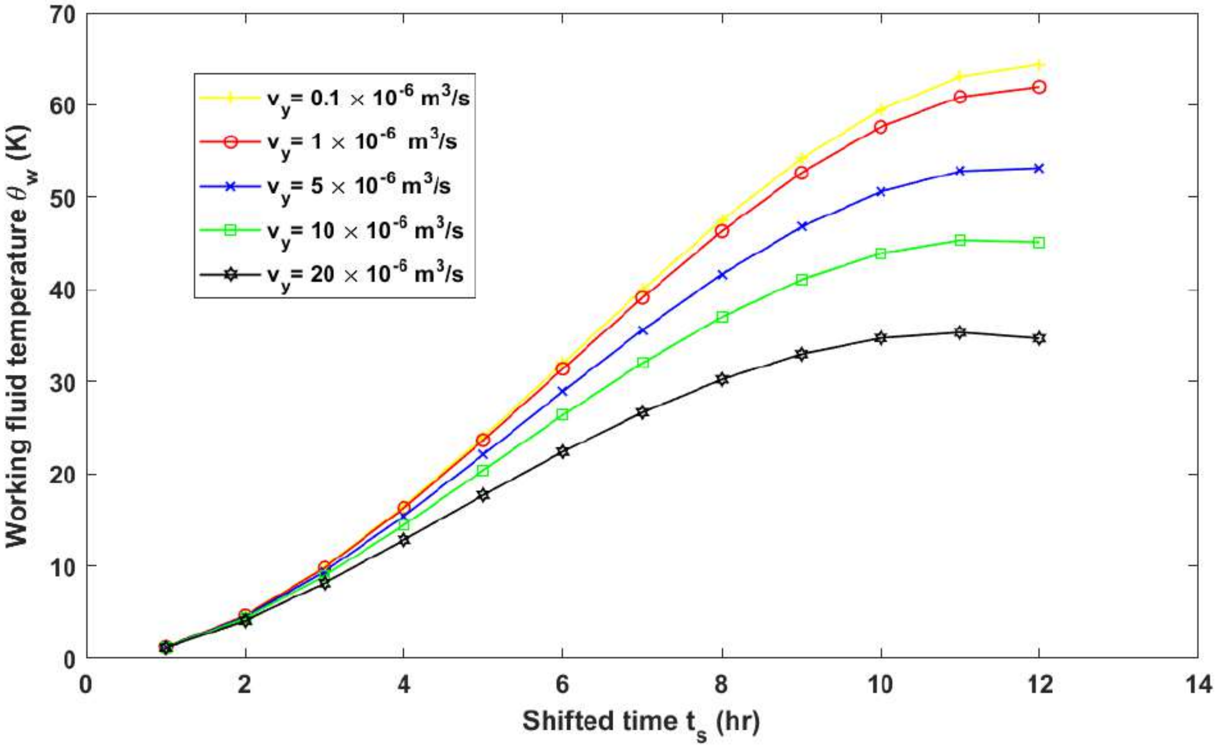

Figure (2): The variation of the temperature of the working fluid $\theta_w(t)$ with the shifted local day time t_s at different flow rates of the fluid

4. Conclusions

The obtained mathematical expressions and computed results reveal the following conclusions, namely:

1- The temperature of the working fluid does depend linearly on the maximum value (q_{max}) of the incident solar irradiance and the absorption coefficient of the absorber material (A) at its front surface.

2- The dependence on the cooling conditions, geometrical, and physical parameters of the absorber Is not linear.

3- The temperature of the working fluid depends weakly on the physical parameters of the absorber material (table 2), this must be taken into consideration for industrial and technological applications.

4- Lower temperatures $\theta_w(t)$ are obtained in general for thicker absorber.

5- The temperature $\theta_w(t)$ increases along the local day time with decreasing the rate of flow and shows asymptotic behavior at the end of the solar day (table 3).

6- The efficiency does depend on the thickness of the absorber and slightly increases with day time. It attains an asymptotic value at the end of the solar day for the considered device (table 4)

7- As cooling (h, W/m^2K) increases, the efficiency increases and it shows asymptotic behavior at the end of the solar day (table 5).

8- As the flow rate of the working fluid increases, the temperature $\theta_w(t)$ decreases. Moreover, at a certain flow rate, the temperature increases with the day time and finally it attains an asymptotic value near the end of the solar day (table 6).

9- As the depth of the reservoir increases, $\theta_w(t)$ decreases.

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