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# How to Manipulate Numerical Bases to Solve Problems

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**Abstract:** The aim of this article is to show the potential of manipulating numerical bases in the solving of problems which involve the sums of integer number powers with equal bases.

## Introduction

The writing of the Hindu-Arabic numbers is very elegant and hides a great treasure, the fact that we write the numbers through the sum of whole powers with the same base.When we write the number 545.3, we have:$5x10^2$ $+4x10$ $+5$ $+$ $3x10^{-1}$ (note that $5x10^0 = 5$). This happens with any base.The number $(471)_{12}$ with base 12 is the same as $4x12^2 + 7x12 + 1$. This less elegant form of writing numbers with any baseshall be fundamental in the understanding of this paper, and will henceforth be referred to as Written by Definition (WD).

Santos[1] used binaries in the WD form to demonstrate the Collatz Conjecture. This shows the great potential of this method of writing whole numbers in mathematical research.

As perthe numerical base concept, supposing the integer "K" is greater than 1,an "n" number can be written in baseK, as follows:

$\mathbf{n} = c_1K^{p1} + c_2K^{p2} +...+ c_nK^{pn}+c$ with $c, c_1,c_2,..c_n< K$

According to Hefez[2] "the positional system is based upon Euclidian division".

In order to change the base of a number, we can use the division algorithm:

$351.25 = 351+0.25$

$0.25\cdot 12 = 3$

351:12= > Q= 29 and R= 3

29:12=> Q= 2 and R = 5

As such, 351.25 in base 12 is 253.3 = $2\cdot 12^2+5\cdot 12+3+3\cdot 12^{-1}$

When using a pencil and paper, the division algorithm can be used as follows:

2 3 5 |5
3 5 47 |5
**0** **2** 9 |5
**4** **1**

Therefore, we have that 235 in base 5 is $(1420)_5$ or $5^3+4x5^2+2x5$ when written using the WD form. Note that for the algorithm,"1" is put as the last quotient, followed by 4, 2 and 0 in a descending order of operations.

In order to facilitate the examples given in this paper,each $c_nK^{pn}$that makes up the number shall be referred to as "Term"(note that they are similar to the terms of a Geometric Progression - GP) and shall be presented in the Written by Definition (WD) format. Following are some example of numbers written in this fashion:

Examples

$2x3^2 + 2x3+1$ (Number in base 3)

$4x5^6+3x5^2+3$ (Number in base 5)

$10x12^2 + 7$ (Number in base 12)

$2^3+2+1$ (Number in base 2)

By definition, when the number is rational, all of its terms' exponents are whole, even of numbers with repeating decimals.

Examples

$-4.5 = -4 -5x10^{-1}$

$20.6666...= 2x10 + 6x10^{-1}+6x10^{-2}+6x10^{-3}+...$

## Where to Use Numbers in the WD Format

For Santos[3], numbers in the WD format can be used in solving problems involving "the summing upof powers of integers with the same base. Therefore, they can be used in Geometric Progressions, Combinatorial Analysis, Number Theory, Exponential Equationsand other problems that have the same characteristics".

WD numbers can solve problems at any educational level, from basic to higher, or in research, provided the problem, be it traditional or original, involves the summing up of powers with the same base, as we shall see in the problems below.

## Exponential Equations

I)Let's find a solution to a question from Paiva's book[4], solving the equation $3^{x+2} + 3^{x-1}= 84$ in IR.

Writing 84 in base 3 using the WD format, we shall have:

$$3^{x+2} + 3^{x-1} = 3^4 + 3$$

Note that $3^{x+2}> 3^{x-1}$ and $3^4> 3$

 

Therefore, we have that $3^{x+2} = 3^4$ and $3^{x-1} = 3$

As such, we shall have two equations: x+2 = 4 and x-1=1.Solving either of the two equations, we end up with x= 2.

II) What is the solution to this equation: $2^{5x} + 2^{3x} + 2^{2x} = 1104$?

Writing 1104 in base 2 using the WD format, we shall have:

$$2^{5x} + 2^{3x} + 2^{2x} = 2^{10} + 2^6 + 2^4$$

Using the same procedures as in the previous problem, we get:

$$2^{5x} = 2^{10},\quad 2^{3x} = 2^6 \text{ and } 2^{2x} = 2^4$$

Solving either of the two equations, we end up with x = 2.

## Geometric Progression

I) Dante[5] (2009,p.148) -How many GP terms (3,6,...) must we consider in order to reach a sum equal to 765?

$$765:3= 255$$

As the ratio is 2, we shall write 255 in base 2 and we get:

$$255 = 2^7+2^6+2^5+2^4+2^3+2^2+2+1 \Rightarrow 7 \text{ terms.}$$

II) Randomly summing up Geometric Progression terms { 4, 12, 36,...} the result was 3348. Which terms were summed up?

Note that the terms of this GP are $4x3^n$and, by definition of numerical bases, the coefficient cannot be greater than the base. As such, prior to writing in base 3, it is necessary to divide by 4 and thusgetting the number 837. Continuing on, we have:

$$837 = 3^6 + 3^4 + 3^3$$

Multiplying each term by 4, we get the terms that had been summed up:

$$4x3^6 = 2916,\ 4x3^4 = 324,\ 4x3^3 = 108$$

Observe that it was also possible to determine that the summed terms were 7º, 6º and 5º.

## Combinatory Analysis

A student summed up all of the non-empty subsets of three sets that had a different quantity of elements and totaled 4129. How many elements were there in each set?

The sum of all the subsets of a set with “n” elements is $2^n$.As the empty set was excluded, we must subtract 1 from each set. As such, we shall have the following equation:

$$(2^x - 1) + (2^y - 1) + (2^k - 1) = 4129$$

$$2^x + 2^y + 2^k = 4129 + 3$$

$$2^x + 2^y + 2^k = 4132$$

$$2^x + 2^y + 2^k = 2^{12} + 2^5 + 2^2$$

Observing the exponents, we see that the sets contain 12, 5 and 2 elements.

## Number Theory

Find a solution withintegers numbers for this equation:

$$5^x + 5^y + 5^k = 750.2$$

Writing 750.2 in the WD format, we get:

$$5^x + 5^y + 5^k = 5^4 + 5^3 + 5^{-1}$$

Therefore, one solution is: x=4, y=3 and k=-1.

## Results

In accordance with the presented results, numerical base manipulation can be an instrument for solving problems that involve sums of the powers of integers with the same base. This allows for the solution of traditional mathematical problems at the basic and higher levels, as well as original problems, making it useful in mathematical research.

In order to manipulate numerical bases, the main instruments are the numbers in WD format (Written by Definition of numerical base). This method of writing numbers allows one to solve problems related to Exponential Equations, Geometric Progressions, Combinatorial Analysis, Number Theory and others of the sort.

## Discussion

Authors like Imenes and Lellis[6], who dedicate themselves to the History of Mathematics or to numbers, always present how numerical bases arose, and that though base 10 is not the most advantageous, it has become the most popular because of its association with the fingers on our hands.

In work done on Number Theory, such as that of Hefez[2], Moreira, Martínez and Saldanha[7], one finds the definition of bases, the Euclidian algorithm and how to change the base of one, as well as some problems particular to bases. Their applicability in other areas of mathematics or science, however, is not presented.

Santos[1][3]follows a completely different path from those of the other authors. He first uses the numerical base definition and numbers in the WD form, to demonstrate theCollatz Conjecture, and then to solve traditional and original problems, such as random sums of geometric progression terms. His work transforms the numerical base into an instrument for investigating and solving mathematical problems. According to the authors, their findings came to light while researching the Collatz Conjecture.

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## Conclusion

Base Number Manipulationis a new instrument in mathematical research with the potential to be used in solving problems from other sciences, as well. As the work done by Santos[1][3] is quite recent, it is necessary for other mathematicians to become interested in the topic so that these instruments can come to be utilized even in other sciences.

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