About the Ultimate Fate of the Universe

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Abstract: *There are different view-points about the ultimate fate of universe. Some of them have emerged from the theory of 'beginning of universe' while, others are the consequences of modern discoveries regarding fundamental particles. What should be the fate of universe is the common subject of these theories. However, what will be the exact fate of universe remains unanswerable. Some theories are relying upon mathematical proofs while, others verify their conceptions by employing observational data. Interestingly, the search for exactness is still in process.*

Keywords: *Big bang theory, lambda CDM model, big rip, dark matter.*

1. INTRODUCTION

Universe is expanding with accelerating rate of expansion.[1] The consequence of expansion or contraction will shape the fate of universe. Many theories proclaim that the present universe is a transitional phase of the bubble universe. [2]The universe existed from very big bang. It might be self-destroyer or it may end as the result of other cosmic happening. Many observations support existence of dark matter. These observations show that dark matter play a significant role in expansion of universe.[3] Upcoming results of particle physics experiment might be brought a new perspective about fate of universe. [4][5] The observational data approves the theory with help of applied mathematics. However, the theories about the ultimate fate of universe are the mathematical guess about the future of the universe. About the theories related to ultimate fate of universe in section 2, some mathematical results according to purposed theory in section 3, the observational data in section4, what should be the fate of universe more precisely in section 5, how mathematics help to understand the exact fate of universe would be in section 6.

2. RELEVANT THEORY ABOUT THE FATE OF UNIVERSE

With the time, discoveries make us more informed about what is going on throughout our universe. Different theory gave us different view with their respective conception. And the results of on-going experiments extract more efficient theory out of them. Here I would like to present those theories, which make our understanding more clear about the fate of universe.

2.1) BIG BANG THEORY: It is a chronicle result of observation of expanding universe. As American Astronomer Edwin Hubble (1929) observed that galaxies are moving away .Which is correlated with their Redshift, [6][7] This theory was standardize by the Lambda CDM model. This model (lambda CDM model) help to describe about the expansion of universe and it is also a root support of big bang theory explaining about expansion of universe. Lambda (ʌ) denote cosmological constant (or anti-gravity component in Einstein's Relativity equation). cosmological constant is a basic component to establish equilibrium in expanding universe, and dark matter described as cold because it's velocity far less than the speed of light at an epoch of radiation-matter equality. However, lambda (ʌ) has push to expand and cold dark matter energy to create matter.[8]

The velocity of expanding universe:

$$v = 2c\left[1-(1+z)^{-1/2}\right] \qquad (1)$$

Where z= Redshift

$$z = [\lambda_{obs} - \lambda_{cm}]\lambda_{em} \qquad (2)$$

$$\begin{pmatrix} \lambda_{obs} = observed\ wavelength \\ \lambda_{em} = emitted\ wavelength \end{pmatrix}$$

& $z = H_0 \frac{D}{C}$ For small distances (3)

Hubble law is a distinct proof of expanding universe. [9]According to Hubble law , the velocity of expanding universe:

$$v = H_0 d \qquad (4)$$

Where H_0 = Hubble constant

All these equations are about calculation to determining the rate of expanding universe. The homogeneous and isotropic universe is expanding with an accelerating rate of expansion.[1] [10] The result of expansion the temperature of universe will have been getting down, as universe achieving maximum size of universe .Consequently two possibilities emerged to deciding the ultimate fate of universe:

i). **Big crunch or Bigfreeze:** One emerging possibility from big bang leads to big crunch, big crunch is a consequence of maximum expanded universe. When the density of expanding universe highly decreases subsequently the metric expansion of space eventually reverse, consequently universe collapse. That phenomenon will have been producing a unified black hole &big crunch singularity. [11]The other possibility is big freeze, as big bang lead expansion the matter density of universe decrease subsequently the temperature will be getting down. The matter density of universe became more than the critical density. The universe started to contract and result would be big crunch, or after expansion going on and on, The temperature of universe will be continuously decrease, the consequence would be big freeze. so fate of universe depend on the origin (big bang) and the way of expansion. [12]

2.2) Big rip with help of dark matter: Big rip theory was proposed in 2003. According to the theory, scale factor of universe will become infinite in finite time with its all distances in universe.[13] It is a possibility which depend on the ratio of pressure and energy density:

$$t_{rip} - t_0 \approx \frac{2}{3|1+\omega|H_0\sqrt{1-\Omega_m}} \quad (5)$$

$$\omega = \frac{P}{\rho} \quad (6)$$

Where P is pressure and ρ is energy density andΩ_m is density of all matter in universe. If the value of ω is closer to -1 then denominator is to be zero and the further big rip will be fate of universe.[14]

According to the latest cosmological data available .there is highly uncertainty in the value of ω, that what is exact value of ω< -1 or ω= -1 or ω > -1. [15] [16]

2.3) Big bounce theory: in the theory of big bounce, the present universe is a phase of expanding or contracting universe. This theory explains about the expansion, as big bang come subsequently after big crunch. Some observations support the theory such as ULAS J1342+0928. See [17] Other evidence with the supporting data. See[18] [19] these all observational data show the periodic expansion and contraction of universe. So big bounce theory is also a relevant theory about the fate of universe.

2.4) Big slurp theory: A new possibility about the fate of universe arises after the conception of Higgs Boson particle. [20] In which Higgs particles interact with other Higgs particle, this process channelize the phenomenon of particle gaining mass more and more. Higgs particle would be bigger, denser and converted into ultra Higgs field. [21] One after one every Higgs convert into a big bubble of mass and they slurp the whole universe. However this process came up with another relevant theory about the fate of universe.

These all chronological conceptions about the ultimate fate of universe are successful corrections in the idea of very big bang of universe. It is very difficult to say that which one is more relevant than other one. Some of them are leading experimental observation while others of them are result of some observational experiment. Here I am trying to present all relevant theory about ultimate fate of universe with observational and experiment evidence very briefly.

3 MATHEMATICAL INTER PRETATION

Now I am trying to show some mathematical interpretation, which are supporting with some extant to the given theory about the fate of universe.

3.1) Future Expansion of the Closed Universe: If the universe is closed, then the total lifetime of the universe, from Big Bang to Big Crunch, can be relatively short in comparison with the characteristic time scales of many of the physical processes. For a closed universe with density parameter Ω_0> 1, the total lifetime τ_U of the universe can be written in the form:

$$\tau_U = \Omega_0(\Omega_0 - 1)^{-\frac{3}{2}}\pi H_0^{-1}, \quad (7)$$

WhereH_0 is the present value of the Hubble parameter [22]. Notice that, by definition, the age $\tau_U \to \infty$ as $\Omega_0 \to 1$. Current cosmological observations suggest that the Hubble constant is restricted to lie in the range 50 – 100 km s^{-1} Mpc^{-1}[23], and hence the time scale $H_0^{-1}$is restricted to be greater than ~ 10 Gyr. Additional observations in [24].Suggest that Ω_0< 2. Using these results, we thus obtain a lower bound on the total lifetime of the universe:

$$\tau_U > 20\pi\ Gyr\ . \quad (8)$$

In terms of the time variable η, this limit takes the form

$$\eta_U > 10.8\ . \quad (9)$$

This limit is not very strong – if the universe is indeed closed, then there will be insufficient time to allow for many of the processes we describe in this paper.

We also note that a closed universe model can in principle be generalized to give rise to an oscillating universe. In this case, the Big Crunch occurring at the end of the universe is really a "Big Bounce" and produces a new universe of the next generation. This idea originated with [25] and has been subsequently considered in many different contexts [26].

3.2) Tunnelling effect may be the future of universe: We next consider the possibility that the universe is currently in a false vacuum state. In other words, a lower energy vacuum state exists and the universe can someday tunnel to that lower energy state. This problem, the fate of the false vacuum, was first explored quantitatively by [27] and by [28]. Additional effects have been studied subsequently, including gravity [29] and finite temperature effects [30].

To obtain quantitative results, we consider an illustrative example in which the vacuum energy density of the universe can be described by the dynamics of a single scalar field. Once a field configuration becomes trapped in a metastable state (the false vacuum), bubbles of the true vacuum state nucleate in the sea of false vacuum and begin growing spherically. The speed of the bubble walls quickly approaches the speed of light. The basic problem is to calculate the tunnelling rate (the decay probability) from the false vacuum state to the true vacuum state, i.e., the bubble nucleation rate P per unit time per unit volume. For tunnelling of scalar fields at zero temperature (generally called quantum tunnelling), the four-dimensional Euclidean action S_4 of the theory largely determines this tunnelling rate. The decay probability P can be written in the form

$$P = Ke^{S_4} \qquad (10)$$

where K is a determinant factor[28] [31]. For purposes of illustration, we assume a generic quartic potential of the form

$$V(\Phi) = \lambda\Phi^4 - a\Phi^3 + b\Phi^2 + c\Phi + d \qquad (11)$$

We can then write the action S_4 in the form

$$S_4 = \frac{\pi^2}{3\lambda}(2-\delta)^{-3}R(\delta)\,, \qquad (12)$$

where $\delta \equiv 8\lambda b/a^2$ and where R is a slowly varying function which has a value near unity for most of the range of possible quartic potentials [32]. The composite shape parameter δ varies from 0 to 2 as the potential V (Φ) varies from having no barrier height to having nearly degenerate vacuum.

Even though equations (10 – 12) describe the tunnelling rate, we unfortunately do not know what potential (if any) describes our universe and hence it is difficult to obtain a precise numerical estimate for this time scale. To get some quantitative feeling for this problem, we consider the following example. For the case of no tunnelling barrier (i.e., for S_4 = 0), the characteristic decay probability is given by $P_0 \sim$ K $\sim M_V^4$, where M_V is the characteristic energy scale for the scalar field. For M_V = 10^{16} GeV (roughly the GUT scale), $P_0 \sim 10^{129} S^{-1} cm^{-3}$. With this decay rate, the universe within a characteristic volume M_V^{-3} would convert from false vacuum to true vacuum on a time scale of $\sim 10^{-24}$ s. Clearly, however, the actual decay time scale must be long enough that the universe has not decayed by the present epoch. In order to ensure that the universe has survived, we require that no nucleation events have occurred within the present horizon volume ($\sim [3000 Mpc]^3$) during the current age of the universe ($\sim 10^{10}$ yr). This constraint implies that the action S_4 must be sufficiently large in order to suppress nucleation, in particular,

$$S_4 > 231 \ln 10 \approx 532\,. \qquad (13)$$

The question then becomes: is this value for S_4 reasonable? For the parameter λ, a reasonable range of values is 0.1 < λ < 1; similarly, for δ, we take the range 0.1 < δ < 1.9. Using the form equation (12) for the action and setting R = 1, we find the approximate range

$$0.5 < S_4 < 3 \times 10^3\,. \qquad (14)$$

Thus, the value required for the universe to survive to the present epoch (equation [14]) can be easily realized within this simple model. In the future, however, the universe could tunnel into its false vacuum state at virtually any time, as soon as tomorrow, or as late as η = 10^4. If and when this tunnelling effect occurs, the universe will change its character almost completely. The physical laws of the universe, or at least the values of all of the physical constants, would change as the phase transition completes (see [33] and [34] for a discussion of changing laws of physics during a future phase transition). The universe, as we know it, would simply cease to exist.

Vacuum tunnelling of the entire universe is certainly one of the more speculative topics considered in this paper. Nevertheless, its inclusion is appropriate since the act of tunnelling from a false vacuum into a true vacuum would change the nature of the universe more dramatically than just about any other physical process.

It is also possible for the universe to spontaneously create "child universes" through a quantum tunnelling process roughly analogous to that considered above [35] [36] [37]. In this situation, a bubble of false vacuum energy nucleates in an otherwise empty space-time. If this bubble is sufficiently large, it will grow exponentially and will eventually become causally disconnected from the original space-time. In this sense, the newly created bubble becomes a separate "child universe". The newly created universe appears quite different to observers inside and outside the bubble. Observers inside the bubble see the local universe in a state of exponential expansion. Observers outside the bubble, in the empty space-time background, see the newly created universe as a black hole that collapses and becomes causally disconnected. As a result, these child universes will not greatly affect the future evolution of our universe because they (relatively) quickly become out of causal contact.

One potentially interesting effect of these child universes is that they can, in principle, receive information from our universe. Before the newly created universe grows out of causal contact with our own universe, it is connected through a relativistic wormhole, which can provide a conduit for information transfer and perhaps even the transfer of matter.

3.3) Entropy and Heat Death: As the fate of universe. The concept of the heat death of the universe has troubled many philosophers and scientists since the mid-nineteenth century when the second law of thermodynamics was first understood. [38] [39][40] Very roughly, classical heat death occurs when the universe as a whole reaches thermodynamic equilibrium; in such a state, the entire universe has a constant temperature at all points in space and hence no heat engine can operate. Without the ability to do physical work, the universe "runs down" and becomes a rather lifeless place. Within the context of modern Big Bang cosmology, however, the temperature of the universe is continually changing and the issue shifts substantially; many authors have grappled with this problem, from the inception of Big Bang theory [41] to more recent times. [42] [43] [44] A continually expanding universe never reaches true thermodynamic equilibrium and hence never reaches a constant temperature. Classical heat death is thus manifestly avoided. However, the expansion can, in principle, become purely adiabatic so that the entropy in a given commoving volume of the universe approaches (or attains) a constant value. In this case, the universe can still become a dull and lifeless place with no ability to do physical work.

Long term entropy production in the universe is constrained in fairly general terms for a given class of systems [45]. For a spatially bounded physical system with effective radius R, the entropy S of the system has a well defined maximum value. This upper bound is given by

$$S \leq \frac{4\pi^2 RE}{hc} \quad (15)$$

where E is the total energy of the system. Thus, for a bounded system (with finite size R), the ratio S/E of entropy to energy has a firm upper bound. Furthermore, this bound can be actually attained for black holes.

In this section I am trying to show that cosmological events continue to produce energy and entropy in the universe, at least until the cosmological decade η ∼ 100. As a result, cosmological heat death is postponed until after that epoch, i.e., until the Dark Era. After that time, however, it remains possible in principle for the universe to become nearly adiabatic and hence dull and lifeless. The energy and entropy generating mechanisms available to the universe depend on the mode of long term evolution. If the universe is closed (above in this section A) or becomes closed at some future time then the universe will end in a big crunch and long term entropy production will not be an issue. For the case in which the universe remains nearly flat, density perturbations of larger and larger size scales can enter the horizon, grow to nonlinearity, and lead to continued production of energy and entropy through the evaporation of black holes (see in appendix 1). These black holes saturate the Bekenstein bound and maximize entropy production. Cosmological heat death can thus be avoided as long as the universe remains nearly flat.

On the other hand, if the universe is open, then density fluctuations become frozen out at some finite length scale (see in appendix 2). The energy contained within the horizon thus becomes a finite quantity. However, the Bekenstein bound does not directly constrain entropy production in this case because the effective size R grows without limit. For an open universe, the question of cosmological heat death thus remains open. For a universe experiencing a future inflationary phase (see in appendix 3), the situation is similar. Here, the horizon is effectively shrinking with time.

However, perturbations that have grown to nonlinearity will be decoupled from the Hubble flow. The largest nonlinear perturbations will, thus define a largest length scale λ and hence a largest mass scale in the universe; this mass scale once again implies a (finite) maximum possible amount of energy available to a local region of space. However, the system is not bounded spatially and the questions of entropy production and cosmological heat death again remain open.
I put forth the point of view that the universe should obey a type of Copernican Time Principle which applies to considerations of the future. This principle holds that the current cosmological epoch (η = 10) has no special place in time. In other words, interesting things can continue to happen at the increasingly low levels of energy and entropy available in the universe of the future.

These all above mathematical interpretation closely support the big crunch as the fate of universe.

4. OBSERVATIONAL DATA: ABOUT ULTIMATE FATE OF UNIVERSE

There is evidence discovered by a group of scientists about the big bang theory of universe. The data received by antenna was sustainable with the theory of big bang. In which, A.A. Penzias and R.W. Wilson noted that The highest frequency at which the background temperature of the sky had been measured previously was 404 Mc/s, where a minimum temperature of 16° K was observed. Combining this value with our result, we find that the average spectrum of the background radiation over this frequency range can be no steeper than$\lambda^{0\ 7}$. This clearly eliminates the possibility that the radiation we observe is due to radio sources of types known to exist, since in this event, the spectrum would have to be very much steeper.[46] this the gateway of new answer about the most celebrated question ,that what will be the ultimate fate of universe. There are two subsequent answer big crunch and big freeze. and some observational support the first one as big crunch will be the ultimate fate of universe. The measured distance-redshift relations of type Ia supernovae (SNeIa) provide the foundation for testing dark energy models. In a flat Universe, the dimensionless luminosity distance

$$d_L(z) = cH_0^{-1}(1+z)\Gamma(z), \quad (16)$$

$$\Gamma(z) = \int_0^z dz'/E(z') \quad (17)$$

Where, $E(z)=E_*(z)/E_*(z=0)$ (18)

The observational data we use in this paper are the same as in [47]. We use the "gold" set of 157 SNeIa (the

Riess sample) published in [48] and analyse it using flux-averaging statistics [49][50] to reduce the bias due to weak gravitational lensing by intervening matter [50]. A Fortran code that uses flux-averaging statistics to compute the likelihood of an arbitrary dark energy model (given the SN Ia data from [48]) can be found at http://www.nhn.ou.edu/~wang/SNcode/ [47][49][50].I only use CMB and LSS data that are not sensitive to the assumptions made about dark energy [47][52].

The only CMB data we use is the measurement of the CMB shift parameter [53],

$$R \equiv \Omega_m^{1/2}\Gamma(z_{CMB}) = 1.716 \pm 0.062 \quad (19)$$

from CMB (WMAP, CBI, ACBAR) [54][55], where z_{CMB}= 1089.

The only large-scale structure information we use is the linear growth rate f(Z_{2dF}) = 0.51± 0.11 measured by the 2dF galaxy redshift survey (2dFGRS) [56], [57], where $z_{2dF} \simeq$ 0.15 is the effective redshift of this survey. Since f = β $*b_1$, where β is the redshift distortion parameter measured from the ratio of the redshift-space correlation function to the real-space correlation function [see Eq.(17) in [56]], and b_1 is the bias factor [square root of the ratio of galaxy power spectrum and mass power spectrum]. Since both correlation function and power spectrum are statistical descriptions of galaxy survey data, they can be extracted from data without making specific assumptions about dark energy. Note that on the other hand, the theoretical prediction of the linear growth rate does depend on assumption about dark energy, as well as cosmological parameters; this is why we can use galaxy survey data to probe dark energy and constrain cosmological parameters. For a given set of cosmological parameters and an assumed dark energy density ρx(z), the linear growth rate f ≡ (dlnD/dlna) is determined by solving the equation for the linear growth rate D,

$$D"(\tau) + 2E(z)D'(\tau) - \frac{3}{2}\Omega_m(1+z)^3 D = 0 \quad (20)$$

Where primes denote d/d(H_0t), and E(z) = $[\Omega_m(1+z)^3 + (1-\Omega_m)\rho x(z)/\rho x(0)]^{1/2}$for a flat universe.

I use the Markov Chain Monte Carlo (MCMC) technique in the likelihood analysis (based on the MCMC engine of [58]), and obtain a few million samples of (Ω_m^*, $\bar{\alpha}$, H_0). This method samples from the full posterior distribution of the parameters, and from these samples the marginalized posterior distributions of the parameters can be estimated. We use the method proposed by Gelman and Rubin to test for convergence [59], [60]. This method uses a convergence indicator

$$\hat{R} = \frac{[(N-1)/N]W + B_n(1+\frac{1}{m})}{W} \quad (21)$$

Where M is the number of chains (each with 2N elements) starting at well-separated points in parameter space, W is the mean variance of the chains, and B_n is the variance between the chains. Convergence is achieved for $\hat{R}$< 1.2. We find that $\hat{R}$< 1.01 for our MCMC chains, which assures us that convergence has been achieved.

In models that lead to a **Big Crunch**, the Hubble parameter H(z) ∝ E(z) will decrease with time until E(z) = 0 at t = t_{turn} , when the universe stops expanding and starts to collapse. Fig.1 of Ref.[61] shows the cosmic scale factor in five models (the linear potential model with five different parameter choices). Clearly, the universe collapses rather quickly after it stops expanding.

Fig.1 shows the constraints on the linear model from current observational data, assuming uniform priors on (ρ_m^0, V_0, α). The first row of figures in Fig.1 shows the probability distribution functions of the set of independent parameters V_0, ρ_m^0 and α. The second row of figures in Fig.1 shows derived parametersΩ_m, h, and the time to collapse from today t_c. The dashed lines denote results using SN data only, the dotted lines denote results using SN together with CMB data, and the solid lines denote results using SN together with CMB and LSS data.

Table 1 lists the mean and the 68% and 95% confidence ranges of Ω_m, h, α, and t_c from Fig.1.

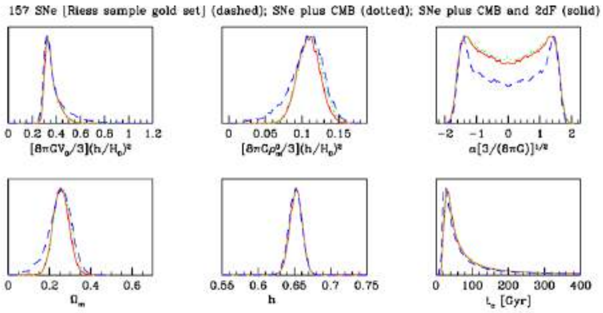

FIG. 1: The constraints on the linear model parameters from current SN data only (Riess sample gold set, fluxaveraged with Δz = 0.05) (dashed lines), SN together with CMB data (dotted lines), SN together with CMB and LSS data (solid lines). Uniform priors on (ρ_m^0, V_0, α) are assumed. The first row shows the probability distribution functions of the set of independent parameters V_0, ρ_m^0 and α The second row shows derived parameters Ω_m, h, and the time to collapse from today t_c.

Table 1: *The constraints on the linear potential dark energy model from current data*

	SN only[a]	SN plus CMB[b]	SN plus CMB and LSS[c]
Ω_m	.25 [.19, .30][.12, .35]	.26 [.22, .30][.18, .35]	.26 [.22, .30][.19, .33]
h^d	.651 [.642, .660][.634, .668]	.651 [.643, .660][.635, .668]	.652 [.644, .661][.635, .668]
$\alpha\left(\frac{3}{8\pi G}\right)^{1/2}$	0.00 [−1.30, 1.30][−1.60, 1.60]	0.00 [−1.20, 1.20][−1.60, 1.60]	0.00 [−1.20, 1.20][−1.60, 1.60]
t_c/[Gyr][e]	56.01 [35.75, —] [19.18, —] [—, 108.18] [—, 2704.01]	67.48 [42.95, —] [23.61, —] [—, 134.34] [—, 4095.18]	65.66 [41.92, —] [23.74, —] [—, 130.52] [—, 3652.01]
χ^2_{min}/N^f_{dof}	25.98/23	26.04 /24	26.16/25

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[a] Riess sample gold set (157 SNe Ia), flux-averaged with $\Delta z = .05$.
[b] CMB shift parameter $R = \Omega_m^{1/2}\Gamma(z_{CMB}) = 1.716 \pm 0.062$.
[c] The linear growth rate $f(z_{2dF}) = 0.51 \pm 0.11$.
[d] Statistical error only, not including the contribution from the much larger SN Ia absolute magnitude error of $\sigma_h^{int} \simeq 0.05$ [14].
[e] For t_c, we list median, the 68% and 95% lower bounds and the 68% and 95% upper bounds.
[f] The number of degrees of freedom.

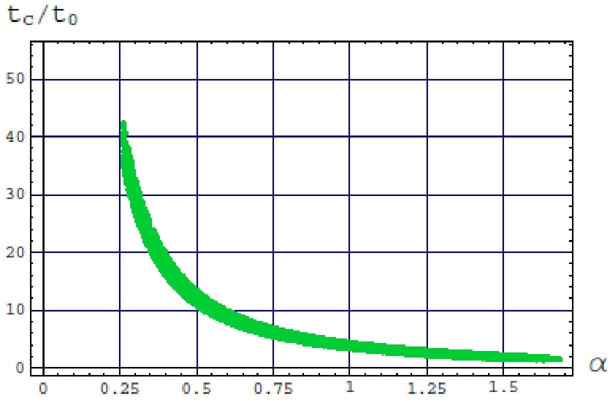

FIG. 2: The ratio of the collapse time from today, t_c, and the age of the universe today, t_0, as a function of the linear potential parameter |α|. Fig.4.

Note that only the median, 68% and 95% confidence lower and upper limits of the collapse time from today t_c are given, since α = 0 is the cosmological constant model with $t_c \to \infty$ (the mean of t_c is not well defined for this reason). Computationally, we have to make a cutoff in tc for all models that are longer lived than the computation limit. The dependence of t_c/t_0 on α is shown in Fig.2, where t0 is the age of the universe today.

Fig.3 shows the effect of assuming different priors for α, using only SN data (Riess sample gold set, flux-averaged with Δz = 0.05). The parameters are the same as in Fig.1. The solid, dashed, and dotted lines correspond to priors of p(α) ∝ 1, $\alpha^{-0.5}$, and α respectively. Table 2 shows how assuming different priors for α changes the median, 68% and 95% lower and upper bounds on the collapse time from today t_c.

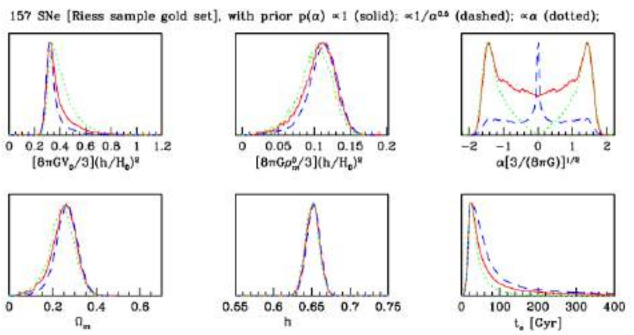

FIG. 3: The effect of assuming different priors for α, using only SN data (Riess sample gold set, flux-averaged with Δz = 0.05). The parameters are the same as in Fig.1. The solid, dashed, and dotted lines correspond to priors of p(α) ∝ 1, $\alpha^{-0.5}$, and α respectively.

Table 2: *Effect of assuming different priors for α on the collapse time from today t_c (in Gyrs)*

$P(\alpha) \propto$	1	$\alpha^{-0.5}$	α
median	56.01	96.92	35.04
68% lower bound	35.75	38.77	27.41
95% lower bound	19.18	19.38	17.91
68% upper bound	108.18	523.35	48.75
95% upper bound	2704.01	714260.81	186.42

These all data are mathematical support to the one of the answer of the fate of universe, and as big freeze is also another subsequent answer of the question. [62]So now we see what will happen with the big freeze singularity:

We shall consider a flat universe which shows a big freeze singularity (BFS) in its future, i.e.

$$\rho = \beta^{-6/\epsilon}\left(a_f^{\epsilon} - a^{\epsilon}\right)^{-3/\epsilon} \tag{22}$$

And

$$p = -c^2 a_f^{\epsilon}\beta^{-6/\epsilon}\left(a_f^{\epsilon} - a^{\epsilon}\right)^{-1-3/\epsilon}, \tag{23}$$

Putting $\epsilon = 3$, we have

$$p = -c^2\beta^3 a_f^3 \rho^2. \tag{24}$$

In this case, one can solve the above equations to find the scale factor. We obtain it in parametric form

$$a = a_f(\sin\eta)^{3/2} \tag{25}$$

$$t = t_f + \frac{1}{k}\left(\ln\left|tan\frac{\eta}{2}\right| + \cos\eta\right), \tag{26}$$

Where

$$k = \frac{1}{\beta}\sqrt{\frac{6\pi G}{a_f^3}}, \qquad dt = \frac{cos^2\eta}{ksin\ \eta}d\eta\ .$$

We set now $t_f = 0$ and therefore η = 0 corresponds to t = −∞, η = π/2 to t = 0 (BFS) and η = π to t = +∞. Now one can see that if ψ = a^3 then the following equation holds

$$\frac{d^2\psi}{dt^2} = (v(t) - \lambda)\psi \tag{27}$$

In which

$$v = \frac{2k^2 sin^2\eta(2cos^2\eta + 1)}{cos^4\eta}, \qquad \lambda = -4k^2.$$

This is very interesting point for the spectral theory of the Schr¨odinger equation. The potential v(t) → +0 at t →±∞ and v(0) = +∞. But we have a bounded state ($\psi \epsilon L^2$ and no zeros at t ∈ (−∞;+∞)), and this is the case notwithstanding for which the potential has a singularity at t = 0 (η = π/2). One can check that

$$\int_{-\infty}^{\infty}\psi^2 dt = 1\,,$$

If $\psi = \sqrt{15k/4}sin^2\eta$

Now we can use Eq. (27) to find the second solution $\hat{\psi}$ with the same potential v and the same value of the spectral parameter λ, i.e.

$\hat{\psi} = \psi \int \frac{dt}{\psi^2}$.

We get

$$\hat{\psi} = -2\cos\eta + \frac{4\cos\eta}{\sin^2\eta} - \sin^2\eta \, \ln\left|\cot\frac{\eta}{2}\right|. \quad (28)$$

Therefore we have a new solution for the same expression

$v - \lambda = 12\pi G\left(\rho - \frac{p}{c^2}\right)$,

This solution describes two universes: the first one begins at t = −∞ and then progressively contracts until a big crunch singularity at t = 0 (or η = π/2). The second solution begins at t = 0 (big bang) and then starts expanding. One can see that the Hubble root$\hat{H} = d\ln\hat{\psi}^{1/3}/dt$ has the asymptotic behavior given by

$$\lim_{t \pm\infty} \hat{H} = \pm 2k\,.$$

and $\hat{H}$ = ∞. Thus we have dS universe at t →±∞.

A most interesting solution is the superposition of ψ = sin^2η and $\hat{\psi}$. We can see that Ψ = $c_1\psi + c_2\hat{\psi}$ results in a new solution $a_{general} = \psi^{1/3}$, such that

$$\frac{d^2\psi}{dt^2} = (v(t) - \lambda)\psi\,. \quad (29)$$

for the same v and λ. This solution describes three distinct kinds of universes. If c_1 = 1 and c_2 =−0.1, then we get Universe I. This universe begins at t = −∞ and then progressively contracts until a big crunch singularity at t = t_i: −∞ <t_i< 0 (for the case c_1 = 1, c_2 = −0.1 we have ti ∼ 0.72).

Universe II. This universe begins at t = t_i (big bang at $a_{general}$ (t_i)= 0) after which it starts expanding until a BFS which takes place at t = 0. It can be shown that this type of universe cannot be fitted to the data of astronomical observations and that it does not lead to a phase of accelerating expansion. Universe III. This universe begins at t = 0 as a **Big freeze singularity** (with $a_{general} \neq 0$ 0) and then starts expanding until it finally behaves like De Sitter for large t

$\lim_{t\to\infty} H_{general} = \frac{2k}{3}$.

the observational data with the mathematical consequence help to establish a relation between the question and answer about the ultimate fate of universe.

5 CALCULATION ABOUT THE AGE OF UNIVERSE

There are theories with mathematical consideration about the age of universe. With help of them we can calculate the time that when the cosmological event will occur:With phantom energy, −(ρ + 3p) increases, and so at some point in time every gravitationally bound system will be dissociated. With the time evolution of the scale factor and the scaling of the phantom-energy density with time, we find that a gravitationally-bound system of mass M and radius R will be stripped at a time t $\simeq P\sqrt{2|1+3\omega|}/[6\pi|1+\omega|]$, where P is the period of a circular orbit around the system at radius R, before the Big Rip (see Table I). Interestingly, this time is independent of H_0 and Ω_m.[63]

TABLE I: The history and future of the Universe with w = −3/2 phantom energy.

As the phantom energy lead universe with continuous expansion. And every object move far and far. Subsequently they stripped apart from each other with the time. And those calculations are here.Big rip may be the answer of the most celebrated question about the ultimate fate of universe with some extent and limitations.[64]

Time	Event
$\sim 10^{-43}$ s	Planck era
$\sim 10^{-36}$ s	Inflation
First Three Minutes	Light Elements Formed
$\sim 10^5$ yr	Atoms Formed
~ 1 Gyr	First Galaxies Formed
~ 15 Gyr	*Today*
$t_{rip} - 1$ Gyr	Erase Galaxy Clusters
$t_{rip} - 60$ Myr	Destroy Milky Way
$t_{rip} - 3$ months	Unbind Solar System
$t_{rip} - 30$ minutes	Earth Explodes
$t_{rip} - 10^{-19}$ s	Dissociate Atoms
$t_{rip} = 35$ Gyrs	Big Rip

6 CONCLUSION

It is probably wrong to say that what will be the exact fate of universe. But with help of observational data, we can guess that what should be the consequence of our expanding universe. It nearly tends to not exactly one solution. It leads to the some solution of exactness, and each of them are significant with some extent with data. But recent discovery solidified the big rip (as dark matter play a relevant role). And the accelerating rate of expansion shows us the fate of universe as big rip. If in future, the rate of expansion will have been slow down the big crunch may be the fate or big bounce .Although, exactly nothing can be done in form of a statement. In the meantime we are intrigued to learn of this possible new cosmic fate that differs so remarkably from the re-collapse or endless cooling. It will be necessary to modify the adopted slogan among cosmic futurologists "Some say the world will end in fire, Some say in ice" [65] — for a new fate may await our world.

Appendixes

Appendix: 1. Continued formation and decay of the black hole: For the case of a flat spatial geometry for the universe, future density perturbations can provide a mechanism to produce entropy. These density perturbations create large structures which can eventually collapse to form black holes. The resulting black holes, in turn, evaporate by emitting Hawking radiation and thus represent entropy (and energy) sources.[66][67] Density perturbations of increasingly larger size scale λ will enter the horizon as the universe continues to expand. The corresponding mass scale M_λ of these perturbations is given by:

$$M_\lambda = M_0\left(\frac{t_\lambda}{t_0}\right)$$

Wheret_λ is the time at which the perturbation enters the horizon and$M_0 \approx 10^{22} M_\odot$ is the total mass within the present day horizon (at time t_0). The time t_λ represents the time at which a given perturbation enters the horizon and begins to grow; a large structure (such as a black hole) can only form at some later time after the perturbation becomes nonlinear. Suppose that a density perturbation has an initial amplitude δ_λ when it enters the horizon. In the linear regime, the perturbation will grow according to the usual relation

$$\delta = \delta_\lambda\left(\frac{t}{t_\lambda}\right)^{2/3},$$

where $\delta \equiv \Delta\rho/\rho$ and t >t_λ [22]. Using this growth law, the epoch η_{nl} at which the perturbation becomes nonlinear can be written in the form

$$\eta_{nl} = \eta_\lambda - \frac{3}{2}\log_{10}\delta_\lambda$$

For example, if the perturbation has an amplitude $\delta_\lambda = 10^{-4}$, then it becomes nonlinear at time $\eta_{nl} = \eta_\lambda + 6$. Since we are interested in very long time scales η > 100, the difference between the horizon crossing time η_λ and the time η_{nl} of nonlinearity is not overly large. One possible result of this process is the production of a large black hole with a mass $M_{BH} \sim M_\lambda$. The time scale for such a black hole to evaporate through the Hawking process is given by

$$\eta_{BH} = 101 + 3\eta_\lambda,$$

the universe can form black holes faster than they can evaporate. Thus, for the case of a geometrically flat universe, future density perturbations can, in principle, continue to produce black holes of increasingly larger mass. In this case, the universe will always have a source of entropy – the Hawking radiation from these black holes.

We note that these bound perturbations need not necessarily form black holes. The material is (most likely) almost entirely non-dissipative and collision less, and will thus have a tendency to form vernalized clumps with binding energy per unit mass of order $\sim\delta c^2$. Thus, unless the perturbation spectrum is tilted so that δ is of order unity on these much larger scales, the ensuing dynamics is probably roughly analogous to that of a cluster-mass clump of cold dark matter in our present universe. However, even if the mass of the entire perturbation does not form a single large black hole, smaller scale structures can in principle form black holes, in analogy to those currently in the centres of present-day galaxies. In addition, it is possible that the existing black holes can merge faster than they evaporate through the Hawking process. Thus, the possibility remains for the continued existence of black holes in the universe. The process outlined here, the formation of larger and larger black holes, can continue as long as the universe remains spatially flat and the density perturbations that enter the horizon are not overly large. The inflationary universe scenario provides a mechanism to achieve this state of affairs, at least up to some future epoch. Thus, the nature of the universe in the far future η ≫ 100 may be determined by the physics of the early universe (in particular, inflation) at the cosmological decade η ∼−45.

Notice that at these very late times, η ≫ 100,Thus, the nucleons will have (most likely) already decayed and the matter content of the universe will be mostly electrons, positrons, and non-baryonic dark matter particles. Annihilation of both e+–e− pairs and dark matter will occur simultaneously with perturbation growth and hence the final mass of the black hole will be less than M_λ. This issue must be studied in further depth.

Appendix: 2. Density Fluctuations and Expansion of a Flat or open universe: The universe will either continue expanding forever or will collapse back in on itself, but it is not commonly acknowledged that observations are unable to provide a definitive answer to this important question. The goal of many present day astronomical observations is to measure the density parameter Ω, which is the ratio of the density of the universe to that required to close the universe. However, measurements of Ω do not necessarily determine the long term fate of the universe.

Suppose, for example, that we can ultimately measure Ω to be some value Ω_0 (either less than or greater than unity). The value of Ω_0 means that the density within the current horizon volume has a given ratio to the critical density. If we could view the universe (today) on a much larger size scale (we can't because of causality), then the mean density of the universe of that larger size scale need not be the same as that which we measure within our horizon today. Let Ω_{big} denote the ratio of the density of the universe to the critical density on the aforementioned larger size scale. In particular, we could measure a value Ω_0< 1 and have Ω_{big}> 1, or, alternately, we could measure Ω_0> 1 and have Ω_{big}< 1. This possibility has been discussed at some length by Linde.[68] [69][70]

To fix ideas, consider the case in which the local value of the density parameter is Ω_0≈ 1 and the larger scale value is Ω_{big} = 2 > 1. (Note that Ω is not constant in time and hence this value refers to thetime when the larger scale enters the horizon.) In other words, we live in an apparently flat universe, which is actually closed on a larger scale. This state of affairs requires that our currently observable universe lies within a large scale density fluctuation of amplitude

$$\frac{\Delta\rho}{\rho} = \frac{\Omega_0 - \Omega_{big}}{\Omega_{big}} = -\frac{1}{2}$$

where the minus sign indicates that we live in a locally under dense region. Thus, a density perturbation with amplitude of order unity is required; furthermore, as we discuss below, the size scale of the perturbation must greatly exceed the current horizon size.

On size scales comparable to that of our current horizon, density fluctuations are constrained to be quite small ($\Delta\rho/\rho \sim 10^{-5}$) because of measurements of temperature fluctuations in the cosmic microwave background radiation.[71][72] On smaller size scales, additional measurements indicate that density fluctuations are similarly small in amplitude.[73][74][75] The microwave background also constrains density fluctuations on scales larger than the horizon [76], although the sensitivity of the constraint decreases with increasing size scale λ according to the relation $\sim(\lambda_{hor}/\lambda)^2$, where λ_{hor} is the horizon size. Given that density fluctuations have amplitudes of roughly ∼ 10−5 on the size scale of the horizon today, the smallest size scale λ_1 for which fluctuations can be of order unity is estimated to be

$\lambda_1 \sim 300\lambda_{hor} \approx 10^6 M_{pc}$.

For a locally flat universe ($\Omega_0 \approx 1$), density fluctuations with this size scale will enter the horizon at a time $t_1 \approx 3\times10^7, t_0 \approx 3\times10^{17}$ yr, or, equivalently, at the cosmological decade

$\eta_1 \approx 17.5$,

This time scale represents a lower bound on the (final) age of the universe if the present geometry is spatially flat. In practice, the newly closed universe will require some additional time to re-collapse and hence the lower bound on the total age becomes approximately η > 18.

The situation is somewhat different for the case of an open universe with $\Omega_0 < 1$. If the universe is open, then the expansion velocity will (relatively) quickly approach the speed of light, i.e., the scale factor will expand according to R ∝ t (for this discussion, we do not include the possibility that $\Omega_0 = 1^{-Q}$, where ϱ ≪ 1, i.e., we consider only manifestly open cases). In this limit, the (comoving) particle horizon expands logarithmically with time and hence continues to grow. However, the speed of light sphere – the distance out to which particles in the universe are receding at the speed of light – approaches a constant in comoving coordinates. As a result, density perturbations on very large scales will remain effectively "frozen out" and are thus prevented from further growth as long as the universe remains open. Because the comoving horizon continues to grow, albeit quite slowly, the possibility remains for the universe to become closed at some future time. The logarithmic growth of the horizon implies that the time scale for the universe to become closed depends exponentially on the size scale λ_1 for which density perturbations are of order unity. The resulting time scale is quite long (η ≫ 100).

To summarize, if the universe currently has a nearly flat spatial geometry, then microwave background constraints imply a lower bound on the total age of universe, η > 18. The evolution of the universe at later times depends on the spectrum of density perturbations. If large amplitude perturbations (Δρ/ρ > 1) enter the horizon at late times, then the universe could end in a big crunch at some time η >η_1 = 17.5. On the other hand, if the very large scale density perturbations have small amplitude (Δρ/ρ ≪ 1), then the universe can continue to expand for much longer time scales. If the universe is currently open, then large scale density perturbations are essentially frozen out.

Appendix: 3. Future inflationary Epochs: We first consider the possibility of a future inflationary epoch. The evolution equation for the universe can be written in the form

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}(\rho_M + \rho_{vac}) \quad \text{(A)}$$

where R is the scale factor, ρ_M is the energy density in matter, and ρ_{vac} is the vacuum energy density. We have assumed a spatially flat universe for simplicity. The matter density varies with the scale factor according to $\rho_M \sim R^{-3}$, whereas the vacuum energy density is constant. We can define the ratio

$v \equiv \rho_{vac}/\rho_0$,

i.e., the ratio of the vacuum energy density to that of the matter density ρ_0 at the present epoch. We can then integrate equation (A) into the future and solve for the time t_{vac} at which the universe becomes vacuum dominated. We find the result

$$t_{vac} = t_0 + \tau\frac{\sinh^{-1}[1] - \sinh^{-1}[v^{1/2}]}{v^{1/2}}, \quad \text{(B)}$$

where t_0 is the present age of the universe and we have defined $\tau \equiv (6\pi G\rho_0)^{-1/2}$; both time scales$t_0$ and τ are approximately 10^{10} yr.

Several results are immediately apparent from equation [B]. If the vacuum energy density provides any appreciable fraction of the total energy density at the present epoch (in other words, if ν is not too small), then the universe will enter an inflationary phase in the very near future. Furthermore, almost any non-vanishing value of the present day vacuum energy will lead the universe into an inflationary phase on the long-time scales. For small values of the ratio ν, the future inflationary epoch occurs at the cosmological decade given by

$\eta_{inflate} \approx 10 + \frac{1}{2}\log_{10}\left[\frac{1}{\nu}\right]$.

For example, even for a present day vacuum contribution as small as $\nu \sim 10^{-40}$, the universe will enter an inflationary phase at the cosmological decade $\eta_{inflate} \approx 30$, long before protons begin to decay. In other words, the traditional cosmological constant problem becomes even more severe when we consider future cosmological decades.

If the universe enters into a future inflationary epoch, several interesting consequences arise. After a transition time comparable to the age of the universe at the epoch , the scale factor of the universe will begin to grow superluminally. Because of this rapid expansion, all of the astrophysical objects in the universe become isolated and eventually become out of causal contact. In other words, every given comoving observer will see an effectively shrinking horizon (the particle horizon does not actually get smaller, but this language has become common in cosmology [77] for further discussion of horizons in this context). In particular, astrophysical objects, such as galaxies and stars, will cross outside the speed-of-light sphere and hence disappear from view. For these same astrophysical objects, the velocity relative to the observer becomes larger than the speed of light and their emitted photons are red shifted to infinity.

REFERENCES

[1] Peebles, P. J. E.; Ratra, Bharat (2003). "The cosmological constant and dark energy". Reviews of Modern Physics. 75 (2): 559–606. arXiv:astro-ph/0207347 Freely accessible. Bibcode: 2003 RvMP...75..559P. doi:10.1103/RevModPhys.75.559.

[2] Tsujikawa, Shinji (28 Apr 2003). "Introductory review of cosmic inflation". (Freely accessible. arXiv:hep-ph/0304257

[3] Trimble, V. (1987). "Existence and nature of dark matter in the universe". Annual Review of Astronomy and Astrophysics. 25: 425–472. Bibcode:1987ARA&A..25..425T. doi:10.1146/annurev.aa.25.090187.002233.

[4] B.C. Allanach ,C.G. Lester, M.A. Parker and B.R. Webber "Measuring sparticle masses in non-universal stringinspired models at the LHC"DAMTP-1999-86Cavendish-HEP-00/06CERN-TH/2000-149

[5] Big Slurp theory about universe (2013) http://cosmiclog.nbcnews.com/_news/2013/02/18/17006552-will-our-universe-end-in-a-big-slurp-higgs-like-particle-suggests-it-might?lite.

[6] Hubble,E.(1929) "A relation between distance and radial velocity among Extra-galactic nanulae". (the proceedings of the national Academy of sciences), vol. 15, no.3.1929.p.p.168-173

[7] Dennis ,overbye(20 February 2017), "cosmos controversy; the universe is expanding, but how fast?" New York Times ,feb.20 2017.https://www.nytimes.com/topic/subject/dark-energy-astronomy

[8] E.J.P. Peebles(1982), "Large scale background temperature and mass fluctuations due to scale-invariant primeval perturbations" the astronomical journal 1982 p.p. 263.

[9] "Hubble Flow". The Swinburne Astronomy Online Encyclopedia of Astronomy. Swinburne University of Technology. Retrieved 2013-05-14.**https://wikivisually.com/wiki/Hubble%27s_law**

[10] Nathaniel Scharping ,(18 oct. 2017)http://blogs.discovermagazine.com/d-brief/2017/10/16/gravitational-waves-universe-expansion/

[11] Kragh,Helge(1996) "cosmology and controversy". Princeton university press.1996, p.p. 318-319

[12] M. Lachieze-Rey, J. PLuminet(1995), "cosmic topology" physics reports 254 (3) ,p.p. 135-214.

[13] Ellis, George F.R., Maartens, roy& Mac Callum, Malcolm A.H. (2012), Relativistic cosmology, p.p. 146- 147.

[14] Robert R. Caldwell, Marc Kamionkowski, Nevin N. Weinberg(2003) "Phantom Energy and Cosmic Doomsday".Journal reference: Phys.Rev.Lett. 91 (2003) 071301, DOI:10.1103/PhysRevLett.91.071301,arXiv:astro-ph/0302506

[15] WMAP has refined its measurements with a final 2 years of data, (sep. 2017), https://map.gsfc.nasa.gov/news/

[16] S.W. Allen , D.A. Rapetti , R.W. Schmidt , H. Ebeling , G. Morris, A.C. Fabian,(2008), "Improved constraints on dark energy from Chandra X-ray observations of the largest relaxed galaxy clusters", monthly notices of royal society of astronomy.Mon.Not.Roy.Astron.Soc.383:879-896, 2008,DOI: 10.1111/j.1365-2966.2007.12610.x, arXiv:0706.0033 [astro-ph]

[17] landau, Elizabeth, Bandos, Eduad (Dec. 2017), "Found: Most Distant Black Hole", https://www.nasa.gov/feature/jpl/found-most-distant-black-hole

[18] Jamie Seide(Dec. 2017), "Black hole at the dawn of time challenges our understanding of how the universe was formed" http://www.news.com.au/technology/science/space/black-hole-at-the-dawn-of-time-challenges-our-understanding-of-how-the-universe-was-formed/news-story/3279356705a47d45416ae2e6ead41175

[19] Bram P. Venemans, Fabian Walter, Roberto Decarli, Eduardo Bañados, Chris Carilli, Jan Martin Winters, Karl Schuster, Elisabete da Cunha, Xiaohui Fan, Emanuele Paolo Farina, (Dec. 2017) "Copious Amounts of Dust and Gas in a z = 7.5 Quasar Host Galaxy". The Astronomical Journal Letters, volume 851, no.1, http://iopscience.iop.org/article/10.3847/2041-8213/aa943a/pdf

[20] Peter w. Higgs, (1964), "Broken symmetries and the masses of gauge Bosons". Physical review Letters , vol-13, no.-16,(19 oct. 1964).

[21] K. Allison, Christopher T. Hill, Graham G. Ross, (2014), "Ultra weak sector, Higgs Boson Mass, and the dilation", Report of FERMILAB , R.no. FERMILAB-PUB-14-086-T,

[22] Peebles, P.J.E. 1993, Principles of Physical Cosmology (Princeton: Princeton Univ. Press).ISBN9780691019338 736 pp. 6 x 9 ¼

[23] Riess, A. G., W. H. Press, and R. P. Kirshner, 1995, Astrophys. J. Lett. 438, L17.

[24] Loh, E., and E. Spillar, 1986, Astrophys. J. Lett. 307, L1

[25] Lemaˆitre, G. 1993, Ann. Soc. Sci. Bruxelles A53, 51.

[26] Tolman, R. C. 1934, Relativity, Thermodynamics, and Cosmology, (Oxford: Clarendon Press).

[27] Voloshin, M. B., I. Yu. Kobzarev, and L. B. Okun, 1975, Sov. J. Nucl. Phys. 20, 644.

[28] Coleman, S. 1977, Phys. Rev. D 15, 2929.

[29] Coleman, S., and F. De Luccia, 1980, Phys. Rev. D 21, 3305.

[30] Linde, A. D. 1983, Nucl. Phys. B216, 421.

[31] Coleman, S. 1985, Aspects of Symmetry (Cambridge, England: Cambridge Univ. Press).

[32] Adams, F. C. 1993, Phys. Rev. D 48, 2800.

[33] Sher, M. 1989, Physics Reports 179, 273.

[34] Crone, M. M., and M. Sher, 1990, Am. J. Phys. 59, 25.

[35] Sato, K., H. Kodama, M. Sasaki, and K. Maeda, 1982, Phys. Lett. 108 B, 103.

[36] Hawking, S. W. 1987, Phys. Lett. 195 B, 337.

[37] Blau, S. K., E. I. Guendelman, and A. H. Guth 1987, Phys. Rev. D 35, 1747.

[38] Helmholz, H. von 1854, On the Interaction of Natural Forces.P.p 489-518 https://doi.org/10.1080/14786445608642114

[39] Clausius, R. 1865, Ann. Physik, 125, 353.

[40] Clausius, R. 1868, Phil. Mag., 35, 405.

[41] Eddington, A. S. 1931, Nature 127, 447.

[42] Barrow, J. D., and F. J. Tipler, 1978, Nature 276, 453.

[43] Barrow, J. D., and F. J. Tipler, 1986, The Anthropic Cosmological Principle (Oxford: Oxford Univ. Press).ISBN-10: 0192821474.

[44] Frautschi, S. 1982, Science 217, 593.

[45] Bekenstein, J. D. 1981, Phys. Rev. D 23, 287.

[46] Penzias, A. A.; Wilson, R. W.(1965) "A Measurement of Excess Antenna Temperature at 4080" Astrophysical Journal, vol. 142, p.419-421 (ApJ Homepage), DOI:10.1086/148307, 1965ApJ...142..419P.

[47] Y. Wang and M. Tegmark, "New dark energy constraints from supernovae, microwave background and galaxy clustering," Phys. Rev. Lett. 92, 241302 (2004) [arXiv:astro-ph/0403292].

[48] A. G. Riess et al. [Supernova Search Team Collaboration], "Type Ia Supernova Discoveries at z > 1 From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution," Astrophys. J. 607, 665 (2004) [arXiv:astro-ph/0402512].

[49] Y. Wang and P. Mukherjee, "Model-Independent Constraints on Dark Energy Density from Flux-averaging Analysis of Type Ia Supernova Data," Astrophys. J. 606, 654 (2004) [arXiv:astro-ph/0312192].

[50] Y. Wang, "Flux-averaging Analysis of Type Ia Supernova Data," Astrophys. J. 536, 531 (2000) [arXiv:astro-ph/9907405].

[51] R. Kantowski, T. Vaughan and D. Branch, "The Effects of Inhomogeneities on Evaluating the Deceleration Parameter q0," Astrophys. J. 447, 35 (1995) [arXiv:astro-ph/9511108]; J. A. Frieman, Comments Astrophys. 18, 323 (1997); J. Wambsganss, R. Cen, G. Xu, J. P. Ostriker, Astrophys. J. 475, L81; D. E. Holz and R. M. Wald, "A New method for determining cumulative gravitational lensing effects in inhomogeneous universes," Phys. Rev. D 58, 063501 (1998) [arXiv:astro-ph/9708036]; R. B. Metcalf and J. Silk, "A fundamental test of the nature of dark matter," Astrophys. J. 519, L1 (1999) [arXiv:astro-ph/9901358]; Y. Wang, "Analytical Modeling of the Weak Lensing of Standard Candles," Astrophys. J. 525, 651 (1999) [arXiv:astro-ph/9901212]; Y. Wang, D. E. Holz and D. Munshi, "A universal probability distribution function for weak-lensing amplification," Astrophys. J. 572, L15 (2002) [arXiv:astro-ph/0204169]; Y. Wang, J. Tenbarge and B. Fleshman, "Simple and Accurate Formulae for Distances in Inhomogeneous Universes," arXiv:astro-ph/0307415.

[52] R. A. Knop et al., "New Constraints on ΩM, ΩΛ, and w from an Independent Set of Eleven High-Redshift Supernovae Observed with HST," Astrophys. J. 598, 102 (2003) [arXiv:astro-ph/0309368].

[53] J. R. Bond, G. Efstathiou and M. Tegmark, "Forecasting Cosmic Parameter Errors from Microwave Background Anisotropy Experiments," Mon. Not. Roy. Astron. Soc. 291, L33 (1997) [arXiv:astro-ph/9702100].

[54] C. L. Bennett et al., "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results," Astrophys. J. Suppl. 148, 1 (2003) [arXiv:astro-ph/0302207]; D. N. Spergel et al. [WMAP Collaboration], "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters," Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209].

[55] T. J. Pearson et al., "The Anisotropy of the Microwave Background to l = 3500: Mosaic Observations with the Cosmic Background Imager," Astrophys. J. 591, 556 (2003) [arXiv:astro-ph/0205388]; C. l. Kuo et al. [ACBAR collaboration], "High Resolution Observations of the CMB Power Spectrum with ACBAR," Astrophys. J. 600, 32 (2004) [arXiv:astro-ph/0212289].

[56] E. Hawkins et al.,(2003) "The 2dF Galaxy Redshift Survey: correlation functions, peculiar velocities and the matter density of the Universe," Mon. Not. Roy. Astron. Soc. 346, 78 [arXiv:astro-ph/0212375];

[57] L. Verde et al.,(2002) "The 2dF Galaxy Redshift Survey: The bias of galaxies and the density of the Universe," Mon. Not. Roy. Astron. Soc. 335, 432 [arXiv:astro-ph/0112161].

[58] A. Lewis and S. Bridle,(2002) "Cosmological parameters from CMB and other data: a Monte-Carlo approach," Phys. Rev. D 66, 103511 [arXiv:astro-ph/0205436].

[59] A. Gelman and D. B. Rubin,(1995) "Inference from Iterative Simulation Using Multiple Sequences," Statistical Science, 7, 457

[60] L. Verde et al., (2003)"WMAP: Parameter Estimation Methodology," Astrophys. J. Suppl., 148, 195

[61] R. Kallosh, J. Kratochvil, A. Linde, E. V. Linder and M. Shmakova, (2003)"Observational Bounds on Cosmic Doomsday," JCAP 0310, 015 [arXiv:astro-ph/0307185].

[62] Artyom V. Yurov and Artyom V. Astashenok, (2007), "Astronomical bounds on future big freeze singularity".Grav.Cosmol.14:205-212,2008 ,DOI:10.1134/S0202289308030018, arXiv:0705.4108 [astro-ph]

[63] McInnes, B.,(2002) "What If w < -1 ?" JHEP 0208, 029 .http://arxiv.org/abs/astro-ph/0210321v1

[64] Robert R. Caldwell, Marc Kamionkowski, Nevin N. Weinberg(2003) "Phantom Energy and Cosmic Doomsday". Journal reference: Phys.Rev.Lett. 91 (2003) 071301, DOI:10.1103/PhysRevLett.91.071301, arXiv:astro-ph/0302506.

[65] Frost, R. "Fire and Ice" (1916), from Collected Poems of Robert Frost, (Holt & Co., 1930).

[66] Page, D. N., and M. R. McKee, 1981a, Phys. Rev. D 24, 1458.

[67] Frautschi, S. 1982, Science 217, 593.

[68] Linde, A. D. 1988, Phys. Lett. 211 B, 29.

[69] Linde, A. D. 1989, Phys. Lett. 227 B, 352.

[70] Linde, A. D. 1990, Particle Physics and Inflationary Cosmology (New York: Harwood Academic).

[71] Smoot, G., et al. 1992, Astrophys. J. Lett. 396, L1.

[72] Wright, E. L., et al. 1992, Astrophys. J. Lett. 396, L3.

[73] Meyer, S. S., E. S. Cheng, and L. A. Page, 1991, Astrophys. J. Lett. 410, L57.

[74] Gaier, T., et al. 1992, Astrophys. J. Lett. 398, L1.

[75] Schuster, J., et al. 1993, Astrophys. J. Lett. 412, L47.

[76] Grischuk, L. P., and Ya. B. Zel'dovich, 1978, Sov. Astron. 22, 125.

[77] Ellis, G.F.R., and T. Rothman, 1993, Am. J. Phys. 61, 883.