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The Arrow of Time

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Abstract:-*There is a perceived dissonance between physical laws which are symmetric in time, and the fact that there seems to be an arrow of time. We analyze two arrows of time: the thermodynamic and the psychological. We provide an explanation of macroscopic irreversibility from the fact that entropy is epistemic not ontic. Irreversibility is therefore not a function of time, but rather probabilistic. We generalize the notion of entropy, and discuss the maximal entropy principle that creates logical irreversibility. The psychological arrow comes from memory, which results from a maximal entropic mechanism. We analyze the memory using a generalized entropy modeland further clarify its statistical nature.*

Keywords: *Entropy, Irreversibility, Second Law, Memory and Maximal Entropy Principle.*

1. INTRODUCTION

Physical laws as we know them do not possess an arrow of time. The laws are time reversal symmetric[1]. Yet in everyday life we clearly experience an arrow of time. We normally see entropy increasing with time, and we can only remember the past, but not the future.

Below, we explain the thermodynamic and psychological arrows of time. Time symmetric physics can explain the perception of both phenomena without directional preference. We will also derive a more general formulation of entropy, as well as an entropic model of memory. In doing so, we will clarify the nature of irreversibility.

2. THERMODYNAMIC ARROW OF TIMEAND LOSCHMIDT'SPARADOX

The second law of thermodynamics is often expressed as that the entropy of an isolated system increases with time monotonically, even though the underlying mechanics is time reversible. This apparent contradiction, known as the Loschmidt's Paradox, can be understood by studying what entropy really represents.

2.1 Entropy and Missing Information

Entropy is often loosely described as a measure of disorder. That lack precise meaning. It is also said that thermodynamics variables are macroscopic while statistical mechanics gives thermodynamics a microscopic foundation. Both statements can be viewed in light of the equivalence of the statistical and the informational entropy.

Consider a classical system with n identical particles. From Newtonian dynamics, the system has $6n$ degrees of freedom, with 3 positional and 3 momentum degrees of freedom for each particle. On the other hand, the number of macroscopic variables available to describe the system (such as $\{n, E, V\}$) is usually much fewer. Therefore, a description of the system (the macrostate) is necessarily missing certain information. Statistical mechanics makes probabilistic inferences about the system using only partial information. The Gibbs entropy $S_G = -k_B \sum_i p_i ln p_i$ arises naturally as a measure of different possible microstates the system is in, under the constraints of $\{n, E, V\}$. By *microstate*, as opposed to the *macrostate*, we mean a full specification of the system in which every degree of freedom is specified with no missing information. Here, n is the number of particles, E is the total energy, and p_i is the probability of each possible microstate i.

Shannon [11] defines the information entropy as $H = -\sum_i p_i log(p_i)$, where p_i is the probability of each possible message i passed through a communication channel. This entropy is a measure of how much informational content any specific message carries, given the space of all possible messages $\{i\}$. It has been pointed out that the Gibbs entropy is only different from the Shannon entropy by a multiplicative factor and the Boltzmann constant k_B, e.g. Jayne [7], or Frigg [4]. This is no surprise, as the statistical mechanic entropy is nothing but the measure of the missing information in the macroscopic

[1] Whether physics is purely time-reversal symmetric as in classical physics and quantum mechanics, or CPT symmetric as in quantum field theory, one can construe physics to be time symmetric. In this paper, we only consider deterministic classic physics or unitary evolution in quantum physics. We do not consider the Copenhagen interpretation of quantum mechanics, which is inherently time-irreversible. The measurement problem poses an entirely different set of philosophical issues which we will not explore.

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thermodynamic description of the physical system. If all $6n$ variables of the microstate are known, that will constitute a complete description (message) of the system consisting of only one microstate. This removes the informational deficiency, and entropy $S = -1 * ln(1) = 0$ will disappear. Since one does not actually know the microstate, but only the macroscopic description, Gibbs entropy measures the information deficiency in such an incomplete description.

This observation suggests that entropy is really an epistemic variable, not an ontic one. By that, we do not mean a sentient being is necessary for entropy to exist. Rather, so long as information about a system is incomplete, entropy exists simply as a measure of the degree of that incompleteness. The magnitude of such missing information is often called disorder or uncertainty. Given a macroscopic description, more possible microstates imply more uncertainty, or a higher entropy. In this paper, we will stick with the more precise terminology of information.

In statistical mechanics, in addition to the known macro variables $\{n, E, V\}$, one can use an additional indicator, i.e, the energy distribution of the particles in the μ-space in the Boltzmann formalism. This distribution can be measured through the system's phenomenological thermodynamic behavior, while the underlying permutation of the particles cannot. Therefore, we ascribe a variable D for the distribution of the n particles to expand the macroscopic description to $\{n, E, V, D\}$, which still entails many possible microstates. Different energy distributions have different entropies associated them, and hence the system's entropy levels can vary. For example, the canonical distribution (classically the Maxwell-Boltzmann distribution) corresponds to the maximum possible entropy for any distribution, and also corresponds to thermal equilibrium.

More accurately, the Gibbs formalism in the γ-space makes explicit that one is not analyzing the microstate itself, but rather an *ensemble* of microstates with the same energy distribution. Geometrically, the definitive microstate of the physical system is a point in the space of descriptive variables, with all $6n$ degrees of freedom specified. On the other hand, the macroscopic description specifies only 3 coordinates, and constitutes a hypersurface covering regions of the γ-space, consisting of many microstates. The system's entropy is defined for the *macrostate*, while the entropy of a microstate (a single point) is by definition zero.

We summarize the distinction between the microstate and the macrostate as follows:

Table 1: Microstate vs. Macrostate

Description	Represents	Entropy
Microstate	physical system	0
Macrostate	ensemble of systems	$-\sum_i p_i ln p_i$
Description	Information	γ-Space Shape
Microstate	Complete	point
Macrostate	Incomplete	hypersurface

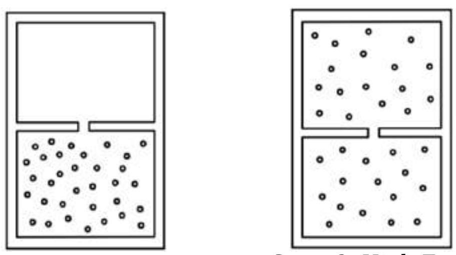
2.2The Second Law and Irreversibility

After setting the table for entropy, we can now interpret the second law properly as a probabilistic statement: at any point in time, it is more likely to find a system in a higher entropy macrostate than in a lower entropy macrostate.

This is different from the traditional version, which asserts that entropy increases absolutely with time, or alternatively - irreversibly with time. It is actually well known that the traditional description of the second law is not always true, but is merely a phenomenological heuristic. There is the well-known fluctuation theorem. Poincare's recurrence theorem further proves that any specific entropy microstate will recur within finite time.

Yet misunderstandings about irreversibility persist in the literature. There is the common refrain that irreversibility is physical and empirically verified, but that statement fails to explain *what* cannot be reversed. Collier [2] also says that reversible mechanics implies that there are as many systems with decreasing entropy as with increasing entropy, and does not quite provide a sufficient justification why reversal is improbable. The basic distinction is the fact that the laws of microstate physics are reversible while the resulting physics of its macroscopic description is irreversible, without any contradictions.

A visual illustration can help. Consider the classical gas chamber. Two microstates are illustrated in the diagrams below:

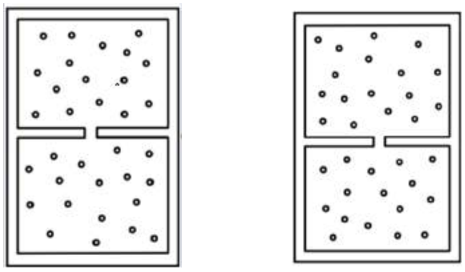


State 1: Low Entropy State 2: High Entropy

Fig -1: Low vs. High Entropy

Typically, we call the State 1 a low entropy state because all particles are concentrated in the lower chamber, and the State 2 a high entropy state because the particles are somewhat evenly distributed between the two chambers. If the separation of the two chambers is removed, one can believe State 1 becomes State 2, but State 2 will not become State 1. Hence, the transition appears to be irreversible.

However, this description is really a hand-waving statement. For example, look at the tow diagrams below (2 or 3). They look similar. However, at a specific future time, State 1 will evolve into one of the two diagrams below, not both. We just do not know which one, unless we follow through the system's evolution over time, with complete specification of all 6n parameters.



Microstate 2 Microstate 3

Fig -2: Different Microstate – Same Macrostate

The problem is loose terminology. When one says that State 1 evolves into State 2 irreversibly, one actually does not imply that the specific Microstate 1 will go to the specific Microstate 2, but rather, "something like Microstate 1 will evolves into something like Microstate 2, IRREVERSIBLY." Or more precisely, the *Macrostate* 1 will evolve into *Macrostate* 2 (which includes both microstate 2 and microstate 3), irreversibly. Using Gibbs terminology, *ensemble* 1 will irreversibly evolve into *ensemble* 2. Because the particles are a lot more likely to be evenly distributed than concentrated, one can expect macrostate 2 will almost never become macrostate 1. It is statistical inference that gives us irreversibility of the macrostate. On the other hand, if we constrain ourselves to only descriptions of the microstate, the transition from microstate 1 to microstate 2 is no more likely than the transition from microstate 2 to microstate 1, hence microstate transitions are completely reversible.

Thus, we conclude: the real distinction between the *microscopic* and *macroscopic* descriptions is not the difference in physical extent, but rather the number of physical microstates (phase space volume) that it entails, or equivalently, the quantity of unknown information as defined by the Shannon entropy. A microstate is a point in the phase space, while the macrostate is a hypersurface with volume. Even though microscopic dynamics (fully specified single-state with no missing information) is reversible, macroscopic dynamics (partially specified ensemble with missing information) is irreversible. A higher entropy macrostate (higher volume hypersurface) is more likely than a lower entropy macrostate (lower volume hypersurface). Since observers inherently cannot observe the microstate, both entropy and irreversibility arise as an epistemic phenomenon.

More rigorously, the physical system (the microstate) engaged in reversible dynamics always has entropy zero regardless of time. On the other hand, its macroscopic description, i.e., the *ensemble* that the microstate corresponds to, which consists of all microstates with the same energy distribution, has varying entropy with time. If the system starts in a microstate that corresponds to a lower entropy macrostate, it *tends* to migrate towards a microstate that corresponds to a higher entropy macrostate at a different time, due only to the fact that the latter is more probable (or has a larger phase space volume). Hence, the microstate is reversible, while the macrostate is irreversible. The fact that empirically, one does see macroscopic irreversible processes in experiments is fully consistent with this view, hence no Loschmidt's paradox.

Several misconstructions about the second law can be settled from this perspective. For example, the problems with Boltzman's flawed H theorem is well known. From our prior discussion, we know that entropy is only *likely* to increase but not always, and hence a theorem that predicts a quantity *always* decreases with time cannot be the right formulation of the second law.

Secondly, the fact that Lousiville's theorem implies Gibbs entropy is constant over time is another source of confusion. Tolman [13] explains it away by using coarse graining of the ensemble γ-space, which does produce a monotonic H with time. However, as Jayne [8] pointed out, that coarse graining is itself an artificial man-made procedure which cannot be the foundation of a true explanation.

The mystery of the constant Gibbs H can be understood as follows. Assuming an initial condition for the system, the phase space volume of its corresponding ensemble stays constant through time. However, the disconnect is this: the microstate of the system is *not* guaranteed to stay in that same ensemble. The second law tells us that the microstate will likely go to a higher entropy macrostate. As that occurs, the ensemble of the system actually changes. When that occurs, the entropy increases as a result. The constancy of the Gibbs H only implies the original ensemble does not change its entropy, but the ensemble representing the system's *macrostate*, actually does change with time. At equilibrium, the entropy is maximized, however, that does not ensure the system staying in the maximum entropy ensemble either, as the fluctuation theorem tells us.

Finally, Jayne's [6] own explanation of the second law is fundamentally flawed. It first equates Gibbs entropy with a concept of experimental entropy, and then introduces a *variation* of the Hamiltonian to the system, which causes the experimental entropy to increase in a new equilibrium, and the constancy of the Gibbs entropy through this process is then said to result in the second law. This is incorrect in that a general entropy increase is supposed to occur under the condition of an *isolated* system, i.e., an *invariant* Hamiltonian. In our description, entropy increase is valid as a probabilistic statement, without a change in the boundary conditions such as the Hamiltonian.

These examples illustrates that when we talk about physical laws, it is imperative not to conflate microscopic physics with the macroscopic. The conservation of the initial ensemble's phase space volume in fact does not imply the system is bound to stay in the same ensemble. Macrostate physics can be *irreversible* while the underlying microstate dynamics is reversible. The nature of irreversibility is probabilistic, not time dependent.

2.3Thermodynamic Arrow of Time

If we assume the system at time $t = 0$ is in a low entropy state L_1, and it evolves as time increases ($t \uparrow$) continuously according to physical laws, with the boundary conditions $\{n, E, V\}$ kept constant, we are more likely to find the system in a higher entropy state L_2 than L_1 at $t > 0$. If the entropy label can be continuously observed, then we will even see a continuous evolution from lower to higher entropy. It applies to any system, and for any entropy label. This is the essence of a proper informational entropic interpretation of the thermodynamic arrow of time.

Note this interpretation works exactly the same way in the opposite time direction ($t \downarrow$) as well. If the boundary conditions are held the same, we are also likely to find the system in a higher entropy state L_2at $t < 0$. Our probabilistic reasoning simply states we are more likely to find it in a higher entropy state at another time, whether at -Δt, or at Δt. Since the reasoning is the same in either direction, witnessing entropy increase does not imply any time direction. Stated differently, time progression in either direction implies entropy increase, but entropy increase does not imply a particular time direction. Hence, the thermodynamic arrow of time does *not* exist. Entropy does not provide a preferred direction of time, or any arrow of time.

In fact, time direction does not even enter into the reasoning. Time symmetry of physical laws (the $t \to -t$ transformation) is a completely different type of argument from the statistical argument of higher entropy. The former is a question of symmetry, while the latter is a question of measure comparison in a metric space. Entropic irreversibility is not the same as time irreversibility. It is all the more apparent if one considers that entropy is always zero when all microstate information is taken into account. Hence, there is no arrow of time. When discussing any system's entropy as a whole, it is only meaningful relative to a specific definition of it. In practice we witness systems going into lower entropy all the time, we just describe them as not isolated.

Finally, when we see a system in a state of low entropy, the real question is how it got there in the first place? As we shall see below, the right answer is usually the tongue-in-cheek one: because someone (or something) put it there. What caused it to be in a low entropic state is that a different boundary condition existed prior, and what was high entropy before, is low entropy now. Therefore, unlike the traditional explanation which is time-asymmetric, our time-symmetric alternative interpretation in the next section will clarify the situation.

3. GENERALIZED ENTROPY AND THE MAXIMAL ENTROPY PRINCIPLE

3.1Generalized Entropy

The previous discussion leads us to generalize the concept of entropy as a measure of the information discrepancy between the system's macroscopic description and its fully specified microstate.

<u>Definition 1: Entropy</u>

When looking at a system, we can separate the descriptive variables of its state into three groups:

- Variables describing the unknown and unobserved degrees of freedom, denoted as $\{x_1 \dots x_f\}$; for future discussions, we assume these degrees of freedom remain unknown
- Variables describing the macro state of the system; we can call them boundary conditions $\{B_1 \dots B_g\}$; often these variables are assumed constant and invariant with time, but this is neither necessary nor always the case; furthermore, these boundary

conditions can be imposed by the environment, and therefore can be construed as "inputs"; for example, the volume of an ideal gas system is set at V

- Lastly, an observable and measurable variable L not imposed by the environment, that further distinguishes among different macroscopically measurable states of the system; it partitions the microstates into mutually exclusive sub-spaces $U_1, U_2 \dots U_L \dots U_m$; the entropy label partitions the phase space; each value of this variable corresponds to a different level of the system's entropy

We call this variable L the entropy label. It should be empirically observable and measurable. Given a specific value of L, we can label the set of microstates with this entropy label L as U_L , and each microstate in the subspace U_L as $i, i = 1,2, \dots$. For a given $L$and U_L, entropy is defined as $S_L = -\int_{i\in U_L} p_i ln p_i$, where each p_i is the conditional probability of a microstate i within the subspace U_L, and other macroscopic variables $\{B_1 \dots B_g\}$.

For future discussions, we will focus on the interaction of this variable with the environment.

This set up matches statistical mechanics discussed above. The boundary conditions $\{B_j\}$ corresponds to the macroscopic variables: total particle number n, total energy E, and maybe total volume V. The entropy label L corresponds to the energy distribution D, which is the canonical distribution at equilibriums. Lastly, our entropy definition matches the Gibbs entropy with a multiplicative constant.

This specification of the entropy label and its emphasis on the role of the information makes it explicit that entropy is epistemic. Our definition of entropy is also different from the traditional one in an important way. Usually it is said that entropy will increase in an *isolated* system, which does not interact with its environment. However, the notion of an isolated system is deceptive. For example, an ideal gas system is always assumed to have boundaries that physically constrain the particles. Therefore, it is really assumed that the particles bouncing off the walls have no significance even as the walls restrict the total volume to V.

Our notation does not make such implicit assumptions of insignificant interactions. These boundary conditions do not imply specific conditions such as zero energy exchanges between the system and the environment. Rather, all relevant information about such exchanges are contained in the boundary conditions $\{B_1 \dots B_g\}$, which do not have to be invariant with time. Under such conditions, the entropy is strictly defined to be a variable about information, even under conditions which are not traditionally considered isolated. This important distinction will be a key to our discussions about memory below.

3.2Maximal Entropy Principle

Given a thermodynamic system with observable macroscopic variables, $\{n, E, V\}$, using a maximum likelihood estimator under the ergodic hypothesis (Tolman [13]), or the maximum entropy principle (Jayne [6]), one can derive the system to be in the canonical energy distribution, which is the Maxwell-Boltzmann distribution in the classical setup.[2]This gives physicists (observers) an additional piece of information, which can be used to make further inferences. Jayne [8] proved that in the canonical distribution the Gibbs entropy is equal to the Clausius thermodynamic entropy: $S_G = S_C = \int \frac{dQ}{T}$. This is how experimental entropy can be measured to validate statistical mechanics.

The concepts of thermodynamic equilibrium and the canonical distributions are intimately related. Under the maximal entropy assumption, the second law indicates the system will likely stay in the same macrostate of highest entropy, which is exactly the definition of a thermodynamic equilibrium. However, we note the maximal entropy condition is really an *assumption*. It is only highly probable, not definitive. In fact, if the system were to always have the maximal entropy, the second law would be meaningless.

Phenomenological thermodynamics has achieved great empirical success under the equilibrium assumption. It allows the processes to be studied as reversible ones (Fermi [3]). Much knowledge was gained from macroscopic phenomenological principles alone. Even the concept of system temperature is only well-defined for thermal equilibriums, known as the zeroth's law of thermodynamics. We note the validity of theory with experiments is not coincidental. The maximally entropic assumption works well when the probability distribution of the system is sharply focused for systems of high degrees of freedom $n \gg 1$.

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2 The reasoning of the quantum entropy in the density matrix von Neumann entropy $-Tr(\rho log \rho)$ is completely analogous because the density matrix formulation assumes the system in a mixed state to have classical probabilities (not unitary transitions in quantum mechanics) associated with each possible pure state, and hence the meaning of the quantum entropy is exactly the same as that of a classical system with many possible microstates.

Jayne [7] may be the first to posit the maximal entropy as a general principle of statistical inference, as a philosophical extension of the principle of insufficient reason. This is not without controversy, e.g., Uffink [14] disputes Jayne's assertion that the Shannon entropy is the unique measure of information based on the consistency principle. However, we do not rely on Jayne's sweeping claim of the maximal entropy as a general principle. Statistical mechanics is based on much more solid physical grounds. The dynamical properties of particle collisions may justify an ergodic assumption about the phase space distribution of microstates. At the same time, the Shannon entropy is *monotonically* increasing with respect to phase space volume (Frigg [4]). This monotonicity property implies a larger ensemble is equivalent to a higher entropy. Our interpretation of the second law is based entirely on an inference on higher probabilities. In situations the ergodic hypothesis is not justified but the probability distribution is still sharply focused, the deductions are still valid. For the rest of the paper, we will use the terms maximal entropy in the sense of the highest probability.

The maximal entropy method is much more widely used in physics than perhaps realized. It is inevitable that we use macroscopic variables to describe systems with many more degrees of freedom. In situations where the maximal entropy state is overwhelmingly more probable than the rest, one can assume the system is indeed in the maximal entropy state (assuming sufficient time has passed), and thus make additional inferences without directly measuring the entropy level. Our discussion of memory below will serve as an example.

Using our generalized entropy notations, we provide an alternative explanation for a system to get into a lower entropy state. Consider the system as defined before. Under one set of boundary conditions $\{B_j\}_1$, the maximal entropy state is L_1; however, if the boundary conditions change to $\{B_j\}_2$, then the maximal entropy state changes to L_2. Why do we witness low entropy states all the time around us, which move to higher entropy states? Because the boundary conditions of that system have changed, what was a maximal entropy state now becomes a low entropy state under the new boundary conditions[3], and we will witness a transition as a result. However, we shall not interpret that the words "prior" and "now", or the past tenses as indicating that there is a time inference here. The exact same reasoning of maximal entropy applies equally well in reverse. If boundary conditions change in the future, and the system's entropy label is known in the future, our inference will be that the maximal entropy state in the past changes accordingly. The informational entropic interpretation is time symmetric.

Essentially, one can construe interpretations in which the environment changes the entropy label by tweaking the boundary conditions. Under the old boundary conditions, the system is presumed to be an old maximal entropy macrostate. By changing the boundary conditions, one can force the system into the new maximal entropy state under the new boundary conditions. With that paradigm in mind, we can now consider the psychological arrow of time.

4. PSYCHOLOGICAL ARROW OF TIME AND THE MAXIMAL ENTROPY PRINCIPLE

The psychological arrow of time refers to the phenomenon that humans view the past as different from the future. For example, we can remember the past, but not the future. It is possible to change the future but not the past. That perception is so deeply ingrained in our interpretive mind, that our perception of cause and effect derives directly from it. Most also believe in free will, and that we can make changes to the future. But free will still prohibits revision of the past. However, no matter whether one believes in free will or not, in any interpretation which distinguishes the past from the future, the assumption of consciousness or free-will is not necessary. It is rather the function of *memory* that is the key element that creates the asymmetric view of time, because for any automaton to effect change on the future based on its experience from the past, memory is required to create that causal link.

4.1Memory and Logical Irreversibility

Let us first examine a common explanation of the psychological arrow of time: for memory to retain information, heat dissipation is required and the total entropy increases for the combined memory and environment. As a result, memory function has to be in the direction of entropy increase, and that must be the same as the thermodynamics arrow of time. This is why the psychological and thermodynamic arrows point in the same direction.

[3] Our description corresponds to the traditional description of entropy change in which a system is not isolated and external influence caused a reduction of entropy.

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However, as we have seen, there is no universal thermodynamic arrow of time, this explanation must be incorrect. If time goes in the reverse direction, life will also see entropy increase for any system starting from a low entropy state. The second law applies equally well in that opposite direction as we discussed. Life may require entropy increases, but that does not explain why memory must go in a specific time direction. If entropy increase is the only distinguishing feature, it is actually perfectly conceivable for one type of life form to remember the past and impact the future, while another type to remember the future and impact the past. Both will see entropy increases.

To answer why memory is asymmetric, one should go back to its basic function. It is a mechanism for recording information, which means a system that can hold *variable* macroscopic states, as a result of its interaction with its environment. The variable macroscopic state will be a quasi-deterministic function of another macroscopic variable from the environment, which we will call "input" from now on.

Rolf Landauer [9] first pointed out that memory operation is *logically* irreversible. Taking a memory system with the minimum information content as an example, something that can switch between a state of 0 and another state of 1, its *logical operation* must be irreversible. Before a recording operation, its state could be either in 0 or 1. But after it records a bit of information from its environment, say 1, then its state is definite. As 1.However, if we look at the device in reverse, starting from a definitive state of 1, its proper operation cannot lead to a bifurcated state with either 0 or 1 as a possibility. Therefore, memory, by definition, cannot be a reversible device. The two sides of a recording operation of the memory thus have to be different. In essence, general purpose memory can only remember one side of time (the past) but not the other side (the future), because that is its purpose and function.[4] No matter what its prior state is, after a recording operation (boundary condition change), it will record a definitive state given by its environment.

4.2An Entropic Model of Memory

We note the thermodynamic irreversibility discussed in Section 3 is exactly the mechanism needed to provide the logical irreversibility needed for memory. There are many possible physical mechanisms for a memory cell, however, all entropic memory systems essentially work in the same way, which can be described as follows:

Definition 2: Memory

Memory is a system with the following properties:

(a) The entropy label L variable stores a piece of information, with different values of L corresponding to different possible values for that information

(b) Under different sets of boundary conditions $\{B_j\}_i$, a different value for the entropic label L_i will have the maximal entropy; the probability of the system failing to be in the maximal entropic state is below a minimal threshold $1 - p(L = L_i)_{\{B_j\}_i} < \varepsilon$

As a result, a different external condition *sets* a different value of the entropic label. This provides the function of memory to record external information from its input. Note, however, that the operation of the memory (setting the cell's state from external input) does not depend on a specific dynamic process, nor is it a function of time. Rather, the operation relies on probabilistic inference, of the maximal entropy kind.

Logical irreversibility is achieved through the entropic irreversibility of the memory system. Consider the scenario below: at time $t \leq 0$, the memory and its boundary conditions could be in any state. Without any loss of generality, we assume it has a value of 0. At time $t = 0$, the input changes to value 1. As the boundary condition for the memory system has changed, the maximal entropy state of the memory becomes 1. However, we know that right at $t = 0_+$, the state of the memory is still 0. It only switches to 1 if a sufficient time delay Δt has elapsed for the new maximal entropy state to materialize. Furthermore, whether the memory was in a state of 1 or 0 at time $t \leq 0$, at $t \geq \Delta t$, the memory state must be at 1 with a sufficiently large probability. Hence, logical irreversibility is achieved. This logical irreversibility actually has time reversal symmetry. If one draws the time arrow in the below diagram from right to left, the same mechanism holds exactly in reverse and the memory will remember the future, instead of the past.

[4] This obviously excludes non-general purpose memories such as quantum coupling which can "remember" the state of a specific coupled particle. However, the fully coupled particle is not the mechanism for memories of life or a general purpose memory cell under current technological conditions.

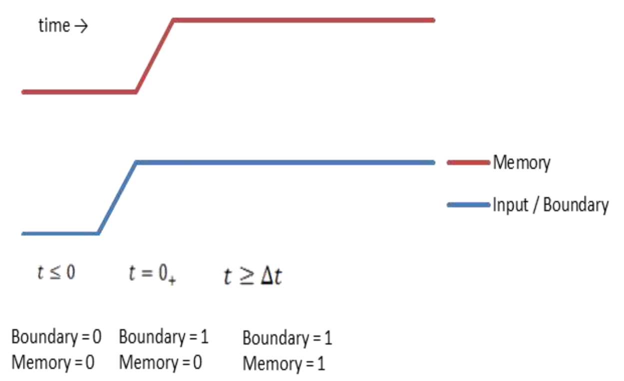

Fig -3: Model of Memory

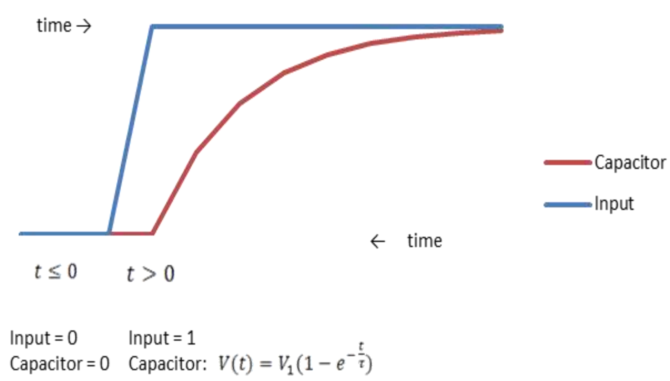

Fig -4: Capacitor Memory

Thus far, we have not resorted to any specific physical mechanism that accomplishes this time-reversible logical irreversibility. The reason is that a macroscopic model of any specific physical mechanism is in reality only an idealized *approximation* of the much more complex underlying microscopic dynamic processes. Such a deterministic model actually hides the fundamental statistical entropic inferential nature of the process. Specifically, such macroscopic deterministic models are usually time-asymmetric which contradicts their underlying time-symmetric microscopic physics. This is often not fully understood or realized by its users.

For example, let us consider a one-bit computer memory cell. If one ignores the details of its positive feedback loop and access mechanisms, the memory cell essentially relies on a capacitor to store a binary state of high voltage (1) or low voltage (0).

The traditional way of looking at capacitor memory is described by the equation:

$$V(t) = V_1\left(1 - e^{\frac{-t}{\tau}}\right) \qquad (1)$$

where$\tau = RC$ and the capacitor can record the fact that voltage changed to V_1, However, this deterministic model runs into a problem upon closer examination. Assume the input and the memory were both at 0 for time $t \leq 0$. At time $t = 0$, the environment flips to 1, and subsequently, the memory also migrates to 1 under equation (1). This makes perfect sense in the forward time direction, as the capacitor responds to the input. However, looking at it from the reverse time direction, we see that while the input voltage is steady at 1, the memory spontaneously decays from 1 to 0. That does not occur in reality. Therefore, the deterministic macroscopic equation is actually *not*reversible, even though the underlying physics at the particle level is time reversible. Therefore, this macroscopic model is indeed only approximate.

What allows irreversibility to take hold is the vast difference in the probabilities of the capacitor to conform to external voltage vs. the capacitor going in the opposite direction. This statistical irreversibility is actually what allows the capacitor to retain the logical irreversibility needed for its memory function.

This situation is exactly analogous to another way to express the second law of thermodynamics: that heat only flows from higher temperature to lower temperature. For example, bringing a heatsink next to a body will cause the body temperature to conform to the heatsink, and the temperature difference will decrease rather than increase. This statement is actually time-irreversible, because looking back in time, heat appears to flow from low temperature to high temperature, and it does not happen. Therefore, the macroscopic statement is time-irreversible while the underlying mechanical collisions of particles are time-symmetric. What we are missing in the deterministic macroscopic statement of direction of heat flow, is that heat flow is a statistical phenomenon, too. The heat conduction equation:

$$q = -k\nabla T \qquad (2)$$

is similarly, only approximate.

The key ingredient for both the capacitor memory and the heat flow to retain macroscopic irreversibility, while the underlying microscopic physic is reversible, is the fact that, the degrees of freedom caused by boundary conditions changes are tremendously more than the degrees of freedom of the system being examined. In the case of the capacitor memory, it is simply *assumed* that there is an infinite degree of freedom from the external voltage source. But in reality the interaction of the capacitor with the voltage source actually causes dissipation and loss in the source (including lowering the input's voltage), which is not accounted for in that equation. The mechanism of voltage source is similar to that of the heat sink, in providing many more degrees of freedom to cause the system to acquire a specific macroscopic state. Electrostatics and thermodynamics just simplifies away the statistical reasoning by assuming infinite degrees of freedom from the external inputs, which allows them to describe the system deterministically as a result.In fact, the heat-conducting body can be construed to be a memory cell itself as well.

Any system that keeps information as a macroscopic state variable will always consist of many potential microscopic states; it is statistical irreversibility, *not* time irreversibility that creates logical irreversibility. Therefore, information replication is only achieved on a statistical basis, not exactly.

4.3Landauer Limit

We can now calculate the entropic implication of memory operations. Within the system itself, the switching of the boundary condition causes the maximal entropic state to switch. If the external condition has equal probability between an 0 or 1, the entropy of either a 0 or 1 state for the memory is the same:

Given $p(B=0)=p(B=1)$, we have

$$S_0 = -\int_{i\in U_0} p_i lnp_i = S_1 = -\int_{i\in U_1} p_i lnp_i \quad (3)$$

But, if there is a definitive external state $B=j$, the maximal entropy state of the memory changes to be the same as the input state, which implies:

$$-\int_{i\in U_j} p_i lnp_{i\,(B=j)} \gg -\int_{i\in U_j} p_i lnp_{i\,(B\neq j)} \quad (4)$$

The logically irreversible memory operation essentially converts from an indefinite state to a definite state, where the memory's probability distribution becomes sharply focused. Before the recording operation, the memory can be in either a 0 or 1 state. After a recording operation, the system is a definitive state, say 1. Assuming the Liouville's theorem, the post recording state of system plus environment occupies the same volume as before the recording in phase space. Since the phase space volume for the system has shrunk by 50% to correspond to a reduction of an indefinite state (both 0 and 1) to a definite state (either a 0 or 1), the phase space volume of the environment must have increased by 100%.

Using the Boltzmann entropy expression with no mixing, the phase space dimensions of the combined system and environment can be separated:

$$S_A = klnW = kln\,(W_S W_E) = klnW_S + klnW_E \quad (5)$$

where the subscript S denotes the system (memory) and the subscript E denotes the environment (input/boundary). In a switch from an indefinite state of 0 and 1 to a definitive state of 1, $\Delta S_A \geq 0$, because of the Second Law of Thermodynamics. The first term $klogW_S$ changes by -$kln2$ because of the 50% shrinkage, therefore the environment entropy $klogW_E$must increase by at least $kln2$. With $dQ = TdS$, we get $dQ_E \geq kTln2$. The Landauer's bound on the minimum heat dissipation for a memory function is thus obtained very simply. This gives a quantitative measure of the entropy, or the degrees of freedom the environment is required to provide for the memory to operate. QED.

5. CONCLUSIONS

In this paper, we have provided an explanation of how time reversible physics can be compatible with the perception that there is an arrow of time. We have formulated generic definitions of entropy and memory. In doing so, we see that entropy is an epistemic variable, that the thermodynamic arrow of time is only relative to an observer who perceives/sets the initial condition of the system with incomplete information. There is no universal thermodynamic arrow. We also see that memory by its nature is irreversible entropy and can derive from time reversible physics. The psychological arrow of time is a maximal entropic statistical phenomenon that comes from changing boundary conditions. We did not discuss the cosmological arrow and why the universe was in such a low entropy state at the big bang. However, this might be a consequence of a change of boundary condition as well.

We do not argue that all physical laws have to be time reversible, but rather that reversible physical laws can explain entropy increases perfectly well.

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