

# Passivity-based Control of Proton Exchange Membrane Fuel Cell Power Generation System

Bing YU<sup>1</sup>, Xianfeng MENG<sup>2</sup>, Yong LUO<sup>3\*</sup>

1. Henan Institute of Metrology, Zhengzhou 450000, P.R.C;

2. China Unicom Zhengzhou Branch, Zhengzhou 450000, P.R.C;

3. School of Electric Engineering, Zhengzhou University, Zhengzhou 450001, P.R.C)

**Abstract:** Control strategy of proton exchange membrane fuel cell (PEMFC) power generating system is studied based on passive theory. Firstly, this paper builds a dynamic mode of PEMFC; Then, the port controlled Hamilton (PCH) model is established based on the equations of State-space averaged model of Boost DC/DC converter and a passive control theory, and a passive controller is designed based on the PCH model and damping injection. Finally, the system simulation model is built based on MATLAB/Simulink, and simulation experiments are done under steady state or disturbances. The simulation results show that the system has good dynamic and steady state characteristics using proposed passive control strategy. These works are helpful for further research of PEMFC power generation system.

**Keywords:** passive control theory; proton exchange membrane fuel cell; Boost DC/DC converter

## 0. INTRODUCTION

Low carbon environmental protection and sustainable development have become modern social development goals in recent years. Distributed generation is becoming an important direction of the sustainable development of power industry, because it can make use of renewable energy well. Now more and more distributed generations including fuel cell, photovoltaic panels, and wind generators, which can generate the power by consuming new or renewable energy are connected to form a big system. Fuel cells have the characteristics of high power generation efficiency, and fuel diversity, and become the most promising new energy power generation technology. Among many kinds of fuel cells, PEMFC draws more attention because of its advantages, such as high power density, low operation temperature, quick start and zero emission<sup>[1-2]</sup>. The problems of slow dynamic

response, unstable output voltage amplitude, and low output power are disadvantages of PEMFC. A DC/DC Boost converter is used in PEMFC of generation electricity system for improving power quality. The voltage conversion can be realized by controlling the duty cycle of PWM signal in DC/DC Boost converter.

Recently, the problem of design a controller to make power not only be stable but also guarantee a quick response of performance has drawn considerable attention. Benchouia et al. (2015) designed an adaptive controller based on the theory of dynamic fuzzy logic<sup>[3]</sup>. The controller can adjust the output voltage of a fuel cell stack according to load change. However, the control method can cause the instability of internal structure of PEMFC, including the rupture of proton exchange membrane and the failure of catalyst. Therefore, it is a more effective method for controlling PEMFC power generation system to control DC/DC Boost converter. Valdivia et al. (2014) analyzed the generation power system including a fuel cell and a DC/DC Boost converter using "black box theory"<sup>[4]</sup>. But the mathematical model covers only the physical nature of the system. The actual physical meanings of the controller's parameters are not clear. Therefore, the research on the control strategy of fuel cell power generation system should consider the physical structure of the system, and the design of the controller should be simple and practical.

So considering comprehensively the above-mentioned factors, theory of passivity based control<sup>[5]</sup> is suitable for the study of control strategies of fuel cell power generation system. The method has been widely used to the control strategies of wind power generation system<sup>[6-7]</sup> and photovoltaic power generation system<sup>[8-9]</sup>. Therefore, this paper studies the control problem of

fuel cell power generation system based on passivity theory. Firstly, this paper presents passivity based on the state-space model of DC/DC Boost converter. Then, the passive control strategy of the power system is proposed. Finally, simulation results are presented to verify the effectiveness of the control strategy.

### 1. SYSTEM DESCRIPTION

The generation system is shown in Figure 1. It is composed of PEMFC stack, DC/DC Boost converter and load.  $u_{FC}$  is the output voltage of PEMFC reactor.  $i_L$  is inductor current.  $L$  is circuit inductance and  $C$  is capacitance.  $R$  is load resistance.  $\mu$  is the duty cycle of main switch.  $u_c$  is the capacitor voltage.

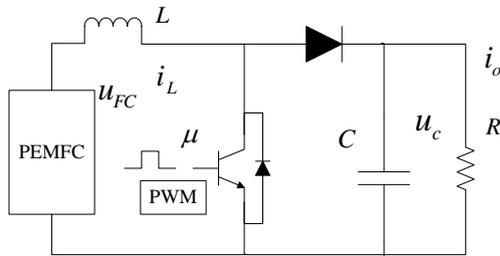


Fig.1. Power circuit of the system

The equivalent circuit of the voltage dynamic model of PEMFC is shown in Figure 2 [10].  $u_{FC}$  is the output voltage of PEMFC.  $E_{Nernst}$  is a Nernst voltage.  $u_{ohm}$  is an activation polarization overvoltage.  $u_{conc}$  is an ohmic polarization overvoltage.  $R_{act}$  is a concentration polarization overvoltage.  $R_{conc}$  is an activation polarization resistance.  $C_{FC}$  is a PEMFC equivalent capacitance.  $u_{CFC}$  is the capacitor voltage.

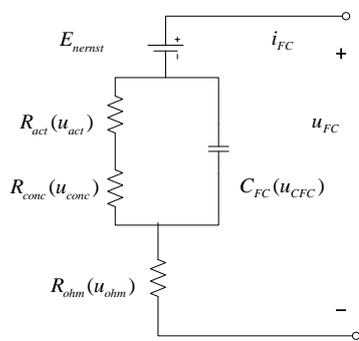


Fig.2. Power circuit of the PEMFC

#### 1.1 PEMFC mathematical model

The basic expression of the voltage for a single fuel cell is [11]:

$$u_{FC} = E_{Nernst} - u_{act} - u_{ohm} - u_{conc} \quad (1)$$

The Nernst equation describing reversible voltage of the cell can be expressed as:

$$E_{Nernst} = \frac{\Delta G}{2F} + \frac{\Delta S}{2F}(T - T_{ref}) + \frac{R_{FC}T}{2F} \left[ \ln(P_{H_2}) + \frac{1}{2} \ln(P_{O_2}) \right] \quad (2)$$

$\Delta G$  is the variation of Gibbs free energy.  $F$  is a Faraday constant.  $\Delta S$  is the variation of Thermodynamics Entropy.  $T$  is fuel cell temperature.  $T_{ref}$  is normal temperature.  $R_{FC}$  is an universal gas constant.  $P_{H_2}$  is the effective pressure of hydrogen.  $P_{O_2}$  is the effective pressure of oxygen.

The activation over potential is

$$u_{act} = -0.9514 + 0.00312T + 7.4 \times 10^{-5} T \ln C_{O_2} - 0.0001877 \ln i_0 \quad (3)$$

$i_0$  is a given current;  $C_{O_2}$  is the concentration of dissolved oxygen at liquid interface, which was defined by the Henry's law:

$$C_{O_2} = P_{O_2} / [5.08 \times 10^6 \exp(-498/T)].$$

The third term of Eq. (1) is an ohmic polarization voltage, It represents the voltage drop due to resistance to the transfer of electrons through the electrodes and to the transfer of protons through the membrane. Using Ohm's law, the expression of the ohmic polarization overvoltage is,

$$u_{ohm} = i_0 (R_M + R_C) \quad (4)$$

Where  $R_M$  is a membrane resistance of the PEMFC.  $R_C$  is a constant.

$$R_M = \rho_M l / A,$$

$$\rho_M = \frac{181.6[1 + 0.03(i_0 / A) + 0.062(T / 303)(i_0 / A)^{25}]}{[\phi - 0.634 - 3(i_0 / A)] \exp[4.18(T - 303) / T]}$$

Where  $l$  is the thickness of proton exchange membrane.  $A$  is the active surface area of proton exchange membrane.  $\rho_M$  is the resistivity of proton exchange membrane.  $\phi$  is the water content of proton exchange membrane.

$$u_{conc} = -B \ln\left(1 - \frac{j}{j_{max}}\right) \quad (5)$$

$B$  is constant.  $j$  is the current density of proton exchange membrane.  $j_{max}$  is the maximum current density.

### 1.2 Mathematical model of DC/DC Boost converter

The main circuit of DC/DC Boost converter is shown in Figure 1. Its state space average model [12] can be expressed as:

$$\begin{cases} L \frac{di_L}{dt} = (\mu - 1)u_c + u_{FC} \\ C \frac{du_c}{dt} = (1 - \mu)i_L - \frac{u_c}{R} \end{cases} \quad (6)$$

Where  $u_{FC}$  is the output voltage of the PEMFC reactor.  $L$  is the circuit's inductance and  $C$  is a capacitance.  $i_L$  is the inductor's current.  $R$  is a load resistance.  $\mu$  is the duty cycle of main switch.  $u_c$  is the capacitor's voltage.

## 2. PASSIVE CONTROLLER DESIGN

### 2.1 Basic knowledge

According to the theory of automatic control [13-14], an system's PCH model can be described as.

$$\begin{cases} \dot{x} = [J(x) - R(x)] \frac{\partial H}{\partial x} + g(x)u \\ y = g^T(x) \frac{\partial H}{\partial x} \end{cases} \quad (7)$$

Where  $H(x)$  is an energy function.  $J(x)$  is a skew symmetric matrix.  $R(x)$  is a semi-definite symmetric matrix.

**Lemma 1:** If the system can be written as the following:

$$M\dot{x} = [\bar{J}(x) - \bar{R}(x)]x + u \quad (8)$$

The system is a PCH system.  $M$  is a symmetric matrix,  $\bar{J}(x)$  is a skew symmetric matrix.  $\bar{R}(x)$  is a semi-definite symmetric matrix.  $x$  is a state vector.  $u$  is an input vector.

**Lemma 2:** If the energy functions of equations (7) and (8) satisfy:  $H(x) \geq 0$ , and satisfy constraint condition:

$$H(x) - H(0) \leq \int_0^t y^T(\tau)u(\tau)d\tau \quad (9)$$

The system is strictly passive.  $u(t)$  is an input vector.  $y(t)$  is an output vector.

### 2.2 Implementation of system PCH model

In order to realize the PCH model of PEMFC power generation system, the state vector and the external input vector of the system are defined as:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} i_L \\ u_c \end{bmatrix}, \quad M = \begin{pmatrix} L & 0 \\ 0 & C \end{pmatrix}, \quad u = \begin{bmatrix} u_{FC} \\ 0 \end{bmatrix}$$

, respectively. Thus, the equation (6) can be written as:

$$M\dot{x} = [\bar{J}(x) - \bar{R}(x)]x + u \quad (10)$$

$$\text{Where, } \bar{J}(x) = \begin{bmatrix} 0 & \mu - 1 \\ 1 - \mu & 0 \end{bmatrix}, \quad \bar{R}(x) = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix},$$

$$\bar{J}(x) = -\bar{J}^T(x), \quad \bar{R}(x) = \bar{R}^T(x) \geq 0. \quad \text{So, from}$$

Lemma 1 the conditions of the implementation of the PCH model are eligible, and equation (10) is the PCH model of the system.

### 2.3 Passivity analysis of PEMFC power generation system

The system's energy function is expressed as:

$$H(x) = \frac{1}{2}x^T Mx = \frac{1}{2}Li_L^2 + \frac{1}{2}Cu_c^2 \quad (11)$$

The derivatives of equations (11) with respect to  $t$  is

$$\dot{H}(x) = Li_L \dot{i}_L + Cu_c \dot{u}_c \quad (12)$$

Let us put equation (6) into equation (12),

$$\dot{H}(x) = i_L u_{FC} - \frac{u_c^2}{R} \leq i_L u_{FC} \quad (13)$$

Substitute  $y = i_L$ ,  $u = u_{FC}$  into equation (13):

$$\dot{H}(x) \leq y^T u \quad (14)$$

By the integration of equation (14) with respect to  $t$ :

$$H(t) - H(0) \leq \int_0^t y^T(\tau)u(\tau)d\tau \quad (15)$$

Therefore, the system expressed by equation (6) is strictly passive and satisfies the design requirements of passive controller according to Lemma 2.

### 2.4 Passive Controller Design

Assume the system stable state vector is expressed as:  $x^* = [x_1^* \quad x_2^*] = [i_L^* \quad u_c^*]$ . System dynamic equation is

$x^* = [x_1^* \quad x_2^*] = [i_L^* \quad u_c^*]$ . System dynamic equation is

$$\begin{cases} L \frac{di_L^*}{dt} = -(1-\mu)u_c^* + u_{FC} \\ C \frac{du_c^*}{dt} = (1-\mu)i_L^* - \frac{u_c^*}{R} \end{cases} \quad (16)$$

From equation (16), the relationship between the inductor current  $i_L^*$  and the output voltage  $u_c^*$  can be obtained as:

$$i_L^* = \frac{u_c^{*2}}{Ru_{FC}} \quad (17)$$

The control problem to be solved in this paper is to design a passive controller to realize the system running in a desired state, namely:  $i_L \rightarrow i_L^*, u_c \rightarrow u_c^*$

Suppose the system error state vector is  $x_e = x - x^*$ . The system error energy storage function can be expressed as:

$$H_e = \frac{1}{2} x_e^T M x_e \quad (18)$$

The controller is designed to make  $H_e$  close to zero.

By differentiating equation (18) with respect to  $t$ :

$$\dot{H}_e = x_e^T M \dot{x}_e \quad (19)$$

Immit the damping  $\bar{R}_a$  into the system [15], namely,  $\bar{R}_a$  is put into equation (10), then we have::

$$M\dot{x}_e = u - Mx^* + \bar{J}(x_e + x^*) - \bar{R}x^* + \bar{R}_a x_e - \bar{R}_d x_e \quad (20)$$

$\bar{R}_d = \bar{R} + \bar{R}_a$ ,  $\bar{R}_a$  is a definite matrix. Suppose  $\bar{R}_a = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$ ,  $r_1, r_2$  are all constants.

Substitute equation (20) into the equation (19):

$$\dot{H}_e = x_e^T [u - Mx^* + \bar{J}(x_e + x^*) - \bar{R}x^* + \bar{R}_a x_e] - x_e^T \bar{R}_d x_e \quad (21)$$

If  $\dot{H}_e \leq 0$ ,  $H_e$  converges to 0. The controller  $u - Mx^* + \bar{J}x - \bar{R}x^* + \bar{R}_a x_e = 0$  can be designed, so that it can ensure  $\dot{H}_e \leq 0$ .

$$u = Mx^* - \bar{J}x + \bar{R}x^* - \bar{R}_a x_e \quad (22)$$

$x^*$  Where  $x^*$  is a constant, so  $M\dot{x}^* = 0$ . Equation (22) can be simplified as:

$$u = -\bar{J}x + \bar{R}x^* - \bar{R}_a x_e \quad (23)$$

Substitute equation (10) into equation (23), two control rates can be get:

$$\mu_1 = 1 - \frac{u_{FC} + r_1(i_L - i_L^*)}{u_c} \quad (24)$$

$$\mu_2 = 1 + \frac{r_2(u_c - u_c^*) - \frac{u_c^*}{R}}{i_L} \quad (25)$$

The DC/DC Boost converter provides power only on power side, that is to say, after the inductor current is increased, the output voltage can be increased, and the expected value of the output voltage can be reached. Therefore, the control rate expressed by equation (25) is ineffective [16]. Based on the above analysis, the control rate of equation (24) is effective for the passive controller designed in this paper.

### 3. SIMULATION EXPERIMENT AND ANALYSIS

In order to verify the effectiveness of the passive controller of the PEMFC power generation system, system simulation model based on MATLAB, including PEMFC, boost DC/DC converter and load is built. PEMFC data is derived from Reference 17. The parameters are shown in Table 1. Fuel cell stack's output voltage is stable at 10 V. DC/DC Boost converter parameters are expressed as:  $L=1\text{mH}$  and  $C=100 \mu\text{F}$ .  $R=40 \Omega$ .

Tab.1 Parameters of the PEMFC

parameters	$T/k$	$A/\text{cm}^2$	$l/\mu\text{m}$	$R_c/\Omega$	$C_{FC}/$
value	353.15	50	51	$\frac{0.000}{3}$	3
parameters	$j_{\max}/\text{A}\square$	$P_{O_2}/0.1\text{M}$	$P_{H_2}/0.1\text{M}$	$\varphi$	$B$
value	1.5	1.5	2	14	0.016

#### 3.1 PEMFC model validation experiments

In order to verify the performance of PEMFC dynamic model, the simulation PEMFC model based on MATLAB is built.

Using a current step input to simulate load change, the current initial value is 5A. In 1st second, the current is

increased to 10A, and the duration is 2 seconds. In 3rd second, the current is reduced to 4A, and the duration is 2 seconds. In 5nd second, the current is increased to 8A. Figure 3 is the output voltage response waveform. Figure 4 is the power response waveform. The output power of the PEMFC stack is expressed as:

$$P_{FC} = u_{FC} \times i_o$$

4. The dynamic model can express the dynamic characteristics of PEMFC exactly, and it builds foundation for the following research.

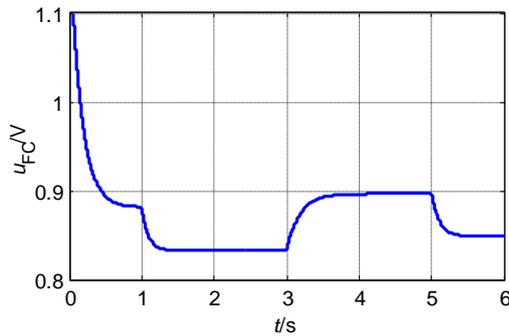


Fig.3. Waveform of the PEMFC stack voltage

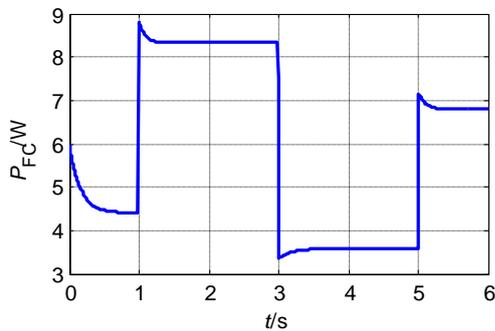


Fig.4. Waveform of the PEMFC stack power

### 3.2 Stable state simulation

The reference value of output voltage is 20V, and the reference value of inductor current is given by equation (17):  $i_L^* = 1A$ . Simulation results are shown

in figure 5 using different damping values,  $r_1$ . It can be seen from figure 3 that at the beginning the PEMFC reactor's voltage rises very fast, and the overshoot is big. However, as shown in figure 5, with the effort of the filter capacitor of DC/DC Boost converter, the output voltage of the system becomes smooth, and the voltage shock is eliminated. The simulation results show that the Boost DC/DC converter that is connected to the

PEMFC system can eliminate the harmonic, so that the output voltage is stable.

Also, as shown in figure 5, the time of the output voltage and the inductor current to the reference value can be changed by adjusting the damping. At the beginning, the change rate of inductor current is faster than that of the output voltage. In transient phase, increasing the damping injection can speed up the response speed of the output voltage, and reduce the overshoot of the inductor current. However, if the damping injection is too much, it will increase the overshoot of the output voltage. Therefore, the damping value should be chosen reasonably.

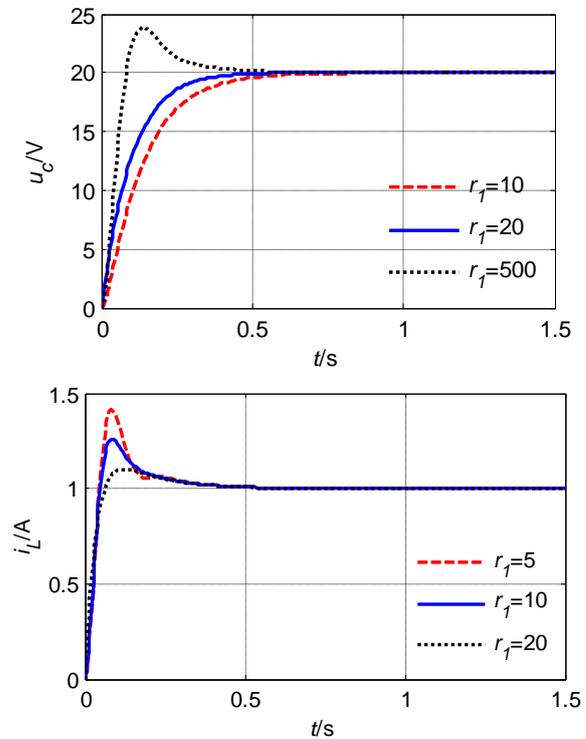


Fig.5. Steady-state waveforms of the system under different values of  $r_1$

### 3.3 Dynamic simulation

#### 3.3.1 Reference current change

The current initial value is 1A, and in 4nd second, the current is increased to 2A, and the duration is 2 seconds. In 6nd second, the current is reduced to 0.5A, and the duration is 2 seconds. In 8nd second, the current is increased to 1 A. The corresponding output voltage reference value is calculated by equation (17):  $20V \rightarrow 28.28V \rightarrow 14.14V \rightarrow 20V$ . The damping value is 20  $\Omega$ . The simulation results are shown in figure 6. It can be seen from figure 4 that the output

voltage and the inductance current can be changed quickly and stably when the reference value of the inductor current is changed.

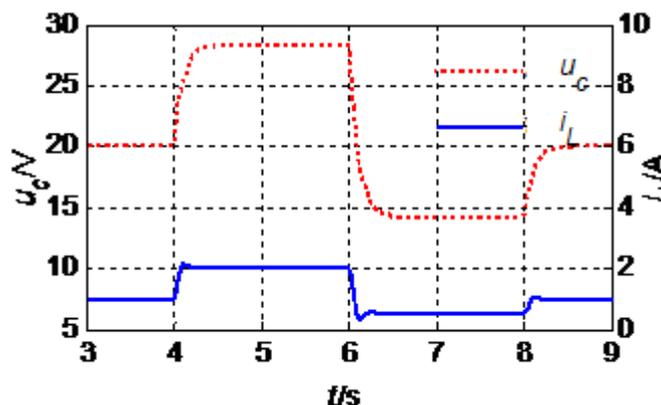


Fig.6. Output response waveform of system with step change of the reference inductance current

### 3.3.2 Load change

The load initial value is  $40\Omega$ , and in 4th second the load is increased to  $80\Omega$ , and the duration is 1 second. In 5th second, the load is reduced to  $40\Omega$ , and the duration is 1 second. In 6th second, the load is reduced to  $20\Omega$ , and the duration is 1 second. In 7th second, the load is increased to  $40\Omega$ . The corresponding inductor current reference value is calculated by equation (17):  $1A \rightarrow 0.5A \rightarrow 1A \rightarrow 2A \rightarrow 1A$ . The damping value is  $20\Omega$ . The simulation results are shown in fig. 7. From Fig. 7, when the load is changed, the output voltage appears small fluctuation, but it is soon recovered to a stable state value, which indicates that the DC/DC Boost converter maintains the output voltage stability by adjusting the inductance current. It is proved that the system has good control performance and ability to resist load disturbance.

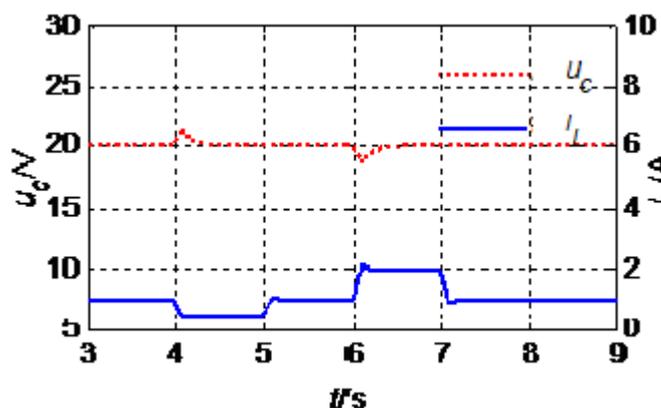


Fig.7. Output response waveform of system under step change of the load

change of the load

## 4. CONCLUSION

In this paper, based on the port controlled Hamiltonian model of DC/DC Boost converter, a passive controller for PEMFC generation system is designed using the theory of passive control. By adjusting the duty cycle of the DC/DC Boost converter, the output voltage of the system can be controlled, and the internal structure of the PEMFC system is not affected. Simulation results show that the system can adjust the steady-state and dynamic performance of the system. Once injected with appropriate damping, the system will have fast tracking ability and the ability to resist load disturbance.

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