

# A Developing of Analytic Inequalities Monograph Based on Students' Mathematical Activities

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**Abstract:-** *The inequalities elaborated in this study are about real and complex numbers. That is an intuitive idea and acquaintance with the properties and used as a background of the study. The more stressing of Analytic Inequalities because of the sophistication, but unsuitable for nearly all undergraduate mathematics education students. Namely, the students are unaccustomed in analyzing the exploitation of inequalities. On terms with inequalities, especially geometric, the students find the concepts as a basis to the inequalities but, less elaborated. In fact, the majority of calculus students are capable of the subject based on the previous learning in the use of inequalities. This mathematics education research is bringing out an elaboration of problemson elementary inequalities dealing with algebra and geometry. The students solve the problems with pencil and paper at hand for amplifying arguments and supplying details and computation. The problems of the students not always ordered in degree of difficulty. Although the students better in coping the concepts of continuity, derivative, and integral, many of them are rather hard. In this case, researchers facilitated them for knowing more easily way to solve the problem. The result is an elaboration monograh that consisted of inequalities in the development of mathematical theory and a glimpse of the analysis. The problems are in following the fundamental rules of algebra and in constructing geometric figures. In proving a problem, the students also need a hypothesis that developed from numerous definitons and postulates or axioms. So, the exercises are available and arranged orderly. Working with the inequalities also required the ability for describing, illustrating and visualizing the problems, especially in a plane.*

**Keywords:** *Inequality, Monograph, Mathematical activity, Algebraic, Geometric*

## 1. INTRODUCTION

The inequalities are statements of the precision of nonelementary real numbers. The elaboration related to the notion of *Pure Mathematics* and *Mathematical Analysis*. Whenever made fundamentally, the property system attended to the fact and deccribed secifically. The

fact recognized in association of the real numbers with points on a straight line. This study based on the growing of larger class of numbers with larger class of points on a line until finally considered to a unique real number. The hard of the inequalities by the students mainly linked to the using of theorem and demonstrated it. Mostly, the truth of an inequalities desired as the validity. For example, using the fact that a number is bigger or small that another one. To show an expression of an inequality using the true and false statement, but derived from the false. For instance, consider the statement  $3 > 4$  and  $1 > -1$ . From this statement,  $9 > 12$  and that  $4 > 3$ . The more sophisticated inequalities established using Principle of Finite Induction. Namely, the inequalities is true for all natural numbers. But, often the inequality will be equivalent to an expression, so that the final solution come to a validity. This reasoning may be reversed and the principle established. For the brevity, the students not always refer to theorems but, recognize the fact that tacitly employed over and over again [1].

### 1.1 Conditional Inequalities

Conditional inequality involving  $n$  variables that does not hold overall of Euclidean  $n$ -space for the entire range. The concept correspond to the distinction between conditional equation and identities in algebra. Another concept in the theory of inequalities is absolute value. That is the distance from a point on the real line to the origin. The students considered the conditional inequalities by geometric interpretations, i.e., to fulfill all points on an open interval. Other strategy to solve the problems is for equivalency of equations. To do this, the students considered cases and come to the consequence. Geometrically, an inequality satisfied on a line.

For a continuous real-valued function defined on a set of a plane, the points describe a curve where the value equal to zero. The curve often divides the plane into a number of subsets through out in holding one of two possibilies of the inequalities.

For that situation, the students must show the plane with regard to the shape. For instance, for a linear function, a region, or a geometric shape.

For complex numbers, a basis consideration is in a triangle. But, the students can also establish the triangle inequalities apart from the geometric consideration. Most of them are that inequalities implied by theorems.

Mathematical activities include the understanding of simultaneous inequalities, locus of points and the relationship between regions formed. The activities contained the bounded area, distance, and equations. That is why the activities far away from calculating, but showing and proving. So, before or after the activities, the students must illustrate an inequality problem visually or geometrically [2].

In studying inequalities, the students come to problems involving calculus concept. For example, isoperimetric problems of all the geometric figures. Some of the problems are extremal, i.e., solved with elementary methods. The methods provide the students an understanding of calculus so not all of the solutions lost.

The isoperimetric problems often more convenient using several quantities. That provided data so easily grasped by the students. The problem is a relation between the algebraic and geometric concepts of the same set of positive numbers [2, 3].

The solutions of the problems provided by geometric, or algebraic observation. After the observation, the students need an application which useful in solving problems. The researcher uses theorems to answer the questions recognized by the students. For example, looking for the surface area of the volume of a solid geometry, the largest volume constructed from a geometric piece by any manipulation, constructing the similarity prove, and thinking analogous.

### 1.2 Isoperimetric

The developing of the monograph also contained isoperimetric that stated for plane geometry figures. There is a difficulty, i.e., the existence of the figures. The consideration is to point out the possibility of a solution of a problem. That is, developing variations on a rigorous basis and establishing conditions under the existence involving mathematics concepts.

The monograph started from a convexity of a plane, specifically the convex bodies. It contains useful theorems and the applications. An elaboration that a plane convex is a closed curve and continuously distorted into the circumference of a circle. The argument showed in Fig- 1 below.

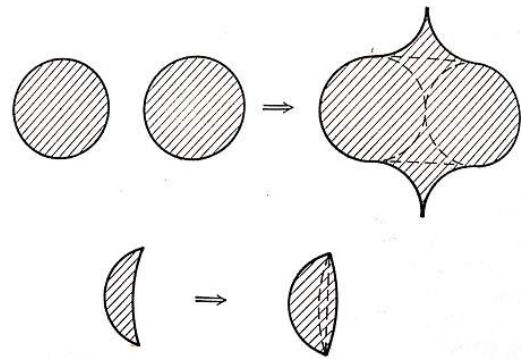


Fig -1: A plane with a perimeter

The figure shows a plane with a perimeter, where if the plane is not convex, the students and the researchers of the class can construct another one with the same perimeter but greater area. For the convex but not a circle, the class also can construct a figure with the same perimeter but with larger area. So, there are possibilities in constructing a figure with a perimeter to another one: (1) the figure is not convex; (2) the figure is convex but not a circle; and (3) the figure is a circle but not convex.

When the figure is not a circle but convex, there are points on its boundary which not a quadrilateral inscribed in a circle. The consideration showed in Fig-2 below.

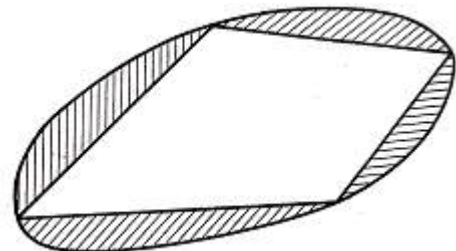


Fig -2: A quadrilateral with flexible joints

The distortion of the quadrilateral into a new one can be inscribed in a circle. Therefore, the total area of the new one plus the attached of not a circle is greater than the first, while the perimeter is the same. Mathematically, if there is some overlapping close to the joints, then the added pieces compensate the lost area but preserved the perimeter.

Actually, the proof of noncircular figure does not follow that the maximum area with a perimeter is exist. For example, there is no figure of least area in which a line segment of a unit length may be so moved as to end up turned around [4].

The students remarked an elementary proof according to the isoperimetric. There are three steps for achieving the proof. Firstly, proved for  $n = 3$  and 4. Secondly, any  $n$ -gon of a perimeter is to have the greatest area of the

regular one. Thirdly, the students prove the isoperimetric theorems.

But, when a perimeter is a nonconvex  $n$ -gon, any sequence is finite and the last is a convex  $k$ -gon. To prove it, the students must use the least upper bound property. The point is that the proof need not be convex and some of its sides lie in its interior as a result of squeezing while a figure with a perimeter converged.

**2. RESULT**

The students’ mathematical activities completed our unique monograph of inequalities. The brief treatment of the topics is no short. Our team know that the elaboration is a systematic study of the mathematics language so the subject provided must be in a specific situation. Regarding to curriculum, this study is to achieve the gaps in textbooks, to explore the subject in the development of mathematical theories, and to explain the lifeblood of mathematical analysis. The content of the manograph consists of fundamentals of algebra or real numbrs, thorems and the derivations, postulate of algbra, lemmas, and ended with some problems. The networking of the contents based on students’ and developers understanding. The depth of the content discussed all together in class. The content presented in Table 1 below.

**Table 1:** Inequalities contents

<i>Fundamental</i>	<i>Theorem</i>	<i>Content</i>
postulates	determinativeness	lemmas
definitions	l.u.b*	corollaries
parameter	g.l.b**	cases
logic		applications

\* l.u.b is least upper bound  
 \*\*g.l.b is greatest lower bound

The table is an explicit elaboration of inequalities, an analytical development in range of mathematics and scientific and engineering fields. The contents presented in accessible language, straight to principles and purposes of mathematical system. The contents show the usefulness of analytical inequalities and apply it to solve problems. The contents used are in real numbers system[5] and complex numbers [6].The contents directed to prove theorems, lemmas, and corollaries.It involves formula manipulation and application to other fields of science.The proof mainly based on explorations [7].Through its thorough, accessible presentation and numerous applications, the monograph of the classic text allows students to work through even the most difficult proofs with ease. It seems to be well demonstrated. The students desire it, then on another occasion intend to a demonstration which will leave no doubt [8].Cases help students test their understanding of the inequalities material at hand and assess their progress through the course.

Numerous cases analyzed based on the actual possibilities. In chemistry, the cases depend on the ideal of physical and chemistry events [9]. For example, an ideal gas described using approximation and inequalities concepts. Another example is about entropy change as a function of reaction of thermodynamic. When using the concepts, the students must organize parametric equation to solve the problem. Another example is the existence of a concept in a vector space that leads to inequalities. The application is in the metric for the single fluid component. All of class activities according to the developing monograph are in Table 2 below.

**Table 2:** Inqualities in the application

<i>Demonstration</i>	<i>Exploration</i>	<i>Proof</i>
illustrating	the range	generalizing
visualizing	identities	by cases
constructing	transformation	equivalency

The generalization is quite useful and enjoyable for all of the class. It’s a great vehicle for discovering new facts. The students always look for new problems after successfully solving one. But, getting more than the expected one to start with the inequalities often come to the difficulty, i.e., lead to erroneous results. The students tests by examples in the sense of validating inequalities problems. They established the characteristic aspect of analytical inequalities and select the postulates and then prove the theorems. But, the expression imposed into the counterbalance the deductives with its development. Invented that, the arguments of the formalistic approach does not reflect on the way of mathematics. Most students validate an idea intuitively and acceptable. However, by discovering a new solution, the unexpected relations lead to further theorems. So, the students always try to find more revealing ways to look at a given statement.The equivalency proof directed to the exploration of inequalities cases. The cases are important because the students often come to incorrect conclusions.The exploration of cases and the illustrations by line numbers raise an understanding of the inequalities concepts. The identities mean look at the range of the inequalities at the same point. While the transformation maps the inequalities onto the plane or figure. The students restrict the solutions that only for the principal value. The activities presented in Chart -1 below.

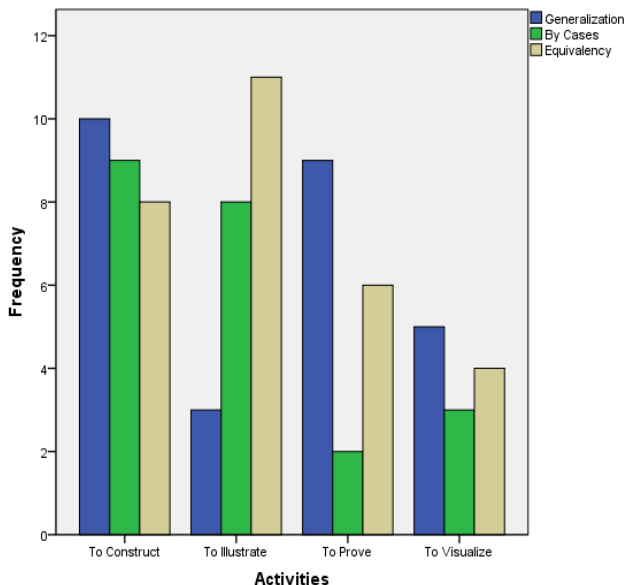


Chart -1: Bar chart of proving activities

The chart illustrates the frequency distribution of the students activities. That is the monograph content resulted through the activities. It shows that looking for equivalent statements is the mostly. Followed by the generalization, and the last one is to prove by cases. But, by regarding to the activities, the students tend to construct the inequalities to easier expression than the others. The second level is to illustrate the inequalities, mainly using numbers and carried out to the validity. The third is to prove the inequalities, mainly using formula that broken down to a simple one. When using the level, the students often need an ability to prove. They use theorems more than lemmas. While the last one is to visualize the inequalities problems. Visualizing the problems actually is important. That is the students want to conclude by specific recognized figure.

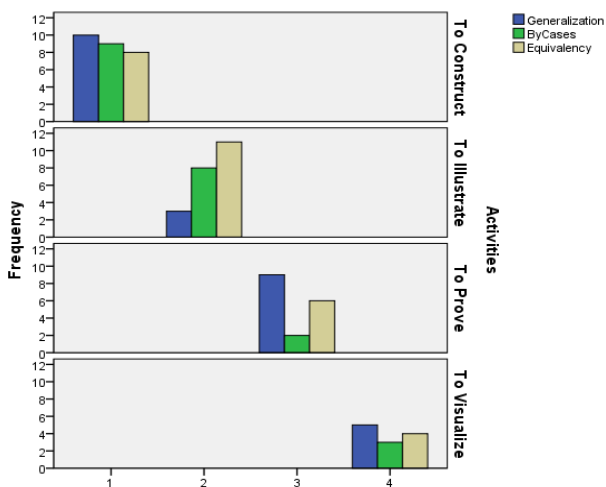


Chart -2: Bar chart of the demonstration

The chart shows that to construct new expression of the inequalities relatively not difference between the three proof activities. But, the order look nice in

developing the monograph. The order is in different direction for illustrating problems. When proving, working by cases is smallest. The demonstration used to develop the monograph based on the students activities. Those described in the list item below, namely the materials, the content, and the activities.

List Item - 1

Problems of a similarly of the principles of real numbers. That is statement to show the inequalities by postulates.

Problems of an implication of a theorem. That is the analysis of the fundamental rules of algebra for inequalities.

Problems of using theorems. That is in constructing more illustrations of the theorems.

Problems of logical construction of postulates. That is the rules in mind to develop numerous meaningful and beautiful inequalities.

Constructing problems. That is the art of presenting inequalities in visual viewpoint.

Demonstrating problems using logic basis. The students activities are working with non-axiomatic logic.

Improving a logical step in a solution. The students activities are in using non-geometric proof.

List Item - 2

Problems of describing a solution area. The students activities are in describing and illustrating the inequalities in a plane.

Problems of a relation between a function and a set of points in a plane. The abilities of the students are in connecting the describing of the graph in the inequalities.

Problems of a distance in complex numbers. That is more students activities in geometric considerations.

List Item - 3

Problems of the simultaneous inequalities of complex plane. That is that the students use a software for comparing and to learn.

Problems of locus of points in a complex plane. The students activities are constructing the manipulative visual of the inequalities.

Problems of geometric and algebraic means. This is important that the inequalities problems started with visual interpretations. The students activities are finding a curve and determining the magnitudes of the geometric quantities.

List Item - 4

Problems of equalities in the inequalities. That is of making estimates or approximations. The students



construct all possibilities natural relationship between algebraic and geometric situations.

Problems of proving theorems geometrically and algebraically. The students collect some cases of the problems carried to an interpretation and the plausibility.

Problems of the product of the positive numbers. The students activities are founding the ways to solve the problems. That is the students' reasoning when use induction principles.

Problems of the product of the positive numbers whose sum is fixed. The problems based on numerous propositions of sets of positive numbers. The students construct a way toward a conjecture.

Problems of applications. The students start a solution of a simple solid geometric problem. The invention is that the using of *Binomial Theorem* and getting a generalization of the inequalities.

### 3. CONCLUSIONS

Mostly, the truth of the inequalities implied to a valid statement. That is the inequalities derived from a fals statement. Therefore, in establishing the inequalities, it's better when using the Principle of Finite Induction. The important concept in the theory of inequalities is of absolute value. The absolute value in the inequalities often comes to equivalent statement. The problems arise because of the visualization of the functions. The students need a figure to observe the phenomena. Another problem is in complex number because of the understanding of positive and negative number. The students understand the problem that observed on logical case. Another understanding based on the numbers, i.e., the examination of the line number. For that problem, the students consider more of distance concept.

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