Chaotic Characteristics of Multi-Dimensional Dynamic System Constructed by Cosine Functions

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Abstract: *Chaos is used to describe and solve non-linear problems, but the current chaos theory and methods are not enough. The chaotic characteristics of existing chaotic systems are not strong enough, and the complex real world can't be better described and predicted by chaotic methods. In this paper, the chaotic characteristics of linear combination of trigonometric functions (cosine functions)are studied. Explore whether the dynamic system iteration of trigonometric functions can be used to better describe the nature of various motions, and give some topics worthy of further study. Finally, image encryption is used to verify the chaotic characteristics of chaotic sequences, which is also an engineering application in computational research field.*

Keywords: *Chaos, Dynamic system, Cosine functions, Image encryption*

1 INTRODUCTION

In recent years, the study of chaos has become a hot topic in the field of non-linear science[1].In some fields, such as fast and low-energy mixing between fluids, chaotic synchronous communication, multimedia information encryption and hiding, fractal graphics drawing in design and art fields, etc.[2], we hope to increase or adjust the chaotic degree of the system for easy application.

2 SYSTEM CONSTRUCTION AND ANALYSIS

In this section, consider the following type of system,

$$\begin{cases} f(x,y) = r_{11}\cos(a_1 x) + r_{12}\cos(b_1 y) \\ g(x,y) = r_{21}\cos(a_2 x) + r_{22}\cos(b_2 y) \end{cases} \quad (1)$$

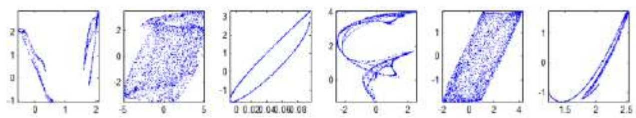
2.1 System Construction

Use Matlab to design the following program, Running the program, generating random numbers as the coefficients of the system, a total of six groups, each group run 1000 times.

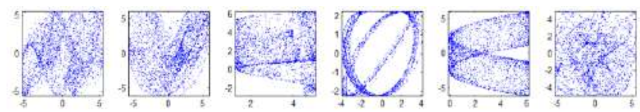
Program1 Drawing Attractors

```
b=rand(10,6)*k;
for j=1:6
subplot(1,6,j)
for i=1:1000
    z1= b(5,j)*cos(b(1,j)*x)+ b(6,j)*cos(b(3,j)*y);
    z2= b(7,j)*cos(b(2,j)*x)+ b(8,j)*cos(b(4,j)*y);
plot(x,y)
hold on
    x=z1;y=z2;
end
axis tight
end
```

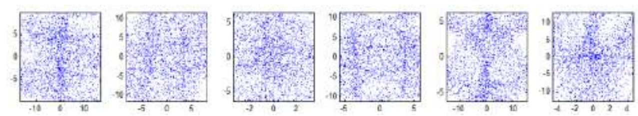
After the program runs, the figure is shown in Figure 1(The parameters are randomly generated and are not recorded here.).



(a) K=2.8



(b) K=3.5



(c) K=6

Figure 1Generating random parameters to generate attractors

2.2 Analysis

When the coefficients are shown in Table 1, they are plotted as shown in Figure 2.

The fourth set of parameters does not produce chaotic attractors because several parameters are too small. a1=0.4287, a2=0.0369, b2=0.6253.

Table 1 *Parameters of six systems(Each system has eight parameters and two initial values)*

	a1	a2	b1	b2	r11	r12	r21	r22	x0	y0
1	1.5592	2.8253	2.5799	2.0250	2.5684	2.0731	1.0738	0.9097	1.0713	1.6770
2	2.2831	0.9712	2.6329	1.7837	1.1631	2.2066	1.7162	1.3969	2.1809	1.9509
3	2.4957	3.0045	1.6409	2.7636	0.5431	3.0764	0.8523	0.7923	2.7498	2.3151
4	0.4287	0.0369	2.8068	0.6253	0.9380	2.0769	0.8930	1.4734	0.2034	3.1034
5	1.8300	1.3298	1.6187	1.0486	1.3593	0.7095	1.8206	2.3875	1.6636	2.0113
6	0.6565	1.1926	2.4597	2.1379	1.4478	1.7830	2.4938	0.1858	1.8930	0.1578

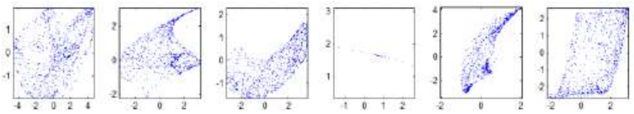


Figure 2 Six attractors

The other parameters remain unchanged, increasing a2 to 1,Attractor is shown in Figure 3. If we fix other parameters, let r11 = 6, r22 = 6,the attractor is shown in Figure 4.

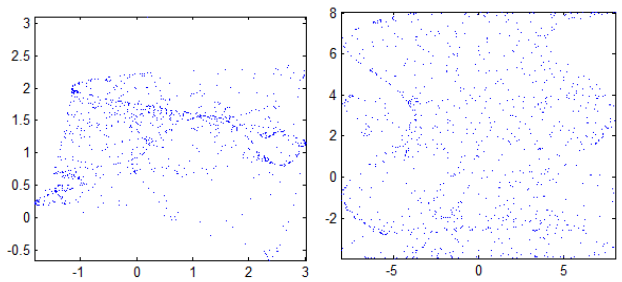


Figure 3 Attractor when a2 = 1Figure 4 r11 = 6, r22=6

The bifurcation diagram of the system varying with a2 is shown in Figure 5.

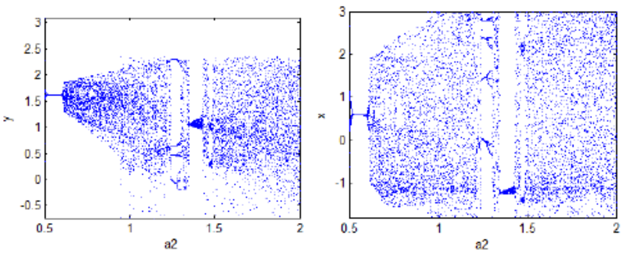


Figure5. *The bifurcation diagram of the system,a2=0.5:0.005:2*

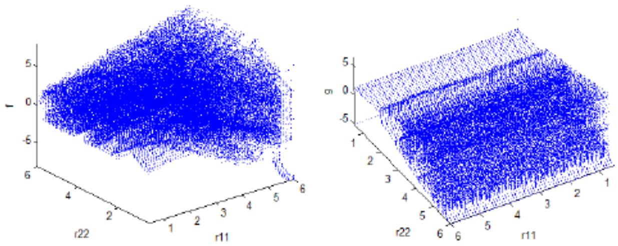


Figure 6 *The bifurcation diagram, r11=0.5:0.1:6, r22=0.5:0.1:6*

3 INCREASED NUMBER OF VARIABLES

Regarding the system shown in Formula (2), program 2 is used to generate it. When the parameters are enlarged to 5, the probability of chaos reaches 99.9% when other random parameters are generated.

$$\begin{cases} f_1(x_1,x_2,\cdots,x_n) = \sum_{i=1}^{n} r_{1i}\cos(a_{1i}x_i) \\ f_2(x_1,x_2,\cdots,x_n) = \sum_{i=1}^{n} r_{2i}\cos(a_{2i}x_i) \\ \cdots \\ f_m(x_1,x_2,\cdots,x_n) = \sum_{i=1}^{n} r_{mi}\cos(a_{mi}x_i) \end{cases} \quad (2)$$

Program 2 Iteration of Multivariate Systems

```
clc; clear all;
a=rand(16,32)*k;
x=rand(1,16); z=zeros(16,2000);
for i=1:2000
for p=1:16
for q=1:32
     z(p,i)=a(p,q)*cos(a(p,q)*x(p))+z(p,i);
end
x(p)=z(p,i);
end
end
```

4 ITERATION WITH IMAGE CONSTRUCTION SYSTEM

The chaotic characteristics of the system shown in Formula (1) are very strong. The dynamic system is composed of f(x,y) and image in formula (1). As shown in Formula (3). When R11 and R12 (which is b(5,j) and b(6,j) in program 1) are greater than 5, the system has the greatest probability of chaos.

$$\begin{cases} f(x,y) = r_{11}\cos(a_1 x) + r_{12}\cos(b_1 y) \\ g(x,y) = Grayscale\ image \end{cases} \quad (3)$$

Figures 7 and 8 below are examples.

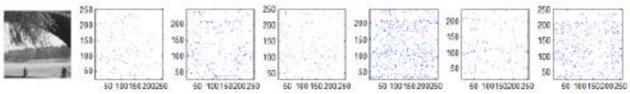


Figure 7f(x,y)= b(5,j)*cos(b(1,j)*x)+ b(6,j)*cos(b(3,j)*y)

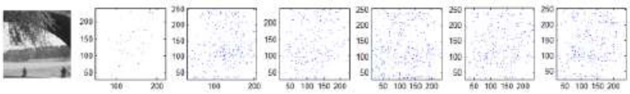


Figure8f(x,y)= b(5,j)*cos(b(1,j)*x)+ b(6,j)*cos(b(3,j)*y)+x/16;

Formula (3) *Whether chaos is also related to images.*

5 CONCLUSION

In this paper, the linear combination of cosine functions is studied, and some programs and illustrations are given. Compared with literature (8) to (11), it is better to generate chaotic sequences (key space) than literature (8) to (11). The content of this paper has not been found in other literatures, and this is a fairly good method for constructing chaotic systems. so it is necessary to study it in depth.

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