International Journal of Innovative Studies in Sciences and Engineering Technology (IJISSET)

ISSN 2455-4863 (Online) **www.ijisset.org** *Volume: 4 Issue: 7 | 2018*

Characterizations for Principal Ideal Graphs of Left Groups and Right Groups

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Abstract*: In this paper, we characterize the principal ideal graphs of a left groups and a right groups. A left group is defined as a direct product of a group and a left zero semigroup. A right group is defined as a direct product of a group and a right zero semigroup. We denote the principal ideal graph of a left group S by ${}_S L_S$. A principal ideal graph of a left group S is the graph whose vertex set is S and any two vertices x and y ($x \neq y$) are adjacent if and only if $xS \cap yS \neq \varnothing$ and $Sx \cap Sy \neq \varnothing$. A principal ideal graph of right group S is defined dually and is denoted by ${}_S R_S$.*

Keywords: *Left groups, Right groups, Principal ideal graphs, Connected graphs, Complete graphs*

1. INTRODUCTION

Algebraic graph theory is a branch of mathematics in which algebraic methods are applied to problems about graphs. The graph theory is very useful in the theory of group and semigroup. The graph can be used to visualize the structure and problems in the semigroup theory. Throughout the past few years, there have been many defined a new type of graphs on semigroups. In 1975, Rosenfeld considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs. The concepts of fuzzy graph has been studied in [9]. In [3] Csakany and Pollak defined intersection graphs of nontrivial proper subgroups of groups. The commuting graphs of rings have been studied in [1] and [2]. The Cayley digraph was defined for the first time by Arthur Cayley in 1878 to study finite groups. Cayley digraphs of semigroups have been studied extensively and many results have been interesting, see for example [5], [6], [7] and [8]. In 2012, the principal ideal graphs of a semigroup was defined by Indu and John in [4]. They studied the principal ideal graphs of a rectangular bands. Inspired by these study, we aim to characterize the principal ideal graphs of a left groups and a right groups.

2. BASIC DEFINITIONS

Let S be a left groups. We define the *principal ideal graph of left group* S as the graph ${}_S L_S$ with S is the vertex set and any two vertices x and y ($x \neq y$) are adjacent in ${}_S L_S$ if and only if $xS \cap yS \neq \varnothing$ and $xS \cap yS \neq \varnothing$. A *principal ideal graph of right group* S is defined dually and is denoted by ${}_S R_S$.

All sets in this paper are assumed to be finite. An element z of a semigroup S is a *left(right) zero* of S if $zs = z(sz = z)$ for all $s \in S$, z is a *zero* of S if it is both a left and right zero of S. A semigroup all of whose elements are left(right) zeros is a *left(right) zero semigroup*. A direct product of a group and a left zero semigroup is called a *left group*. A direct product of a group and a right zero semigroup is called a *right group*. The *cardinality* of a set X, denoted by $|X|$, is the number of elements in X. For any family of nonempty set $\{X_i \mid i \in I\}$, let $\dot{\bigcup}_{i \in I} X_i$ denote the disjoint union of X_i, $i \in I$.

Let G_1 and G_2 be graphs. A mapping $\varphi : V(G_1) \to V(G_2)$ is called a *graph homomorphism* if any two vertices u and v of G_1 are adjacent in G_1 implies $\varphi(u)$ and $\varphi(v)$ are adjacent in G_2, i.e. φ preserves edge. We write $\varphi : G_1 \to G_2$. If $\varphi : G_1 \to G_2$ is a bijective graph homomorphism and φ^{-1} is also a graph homomorphism, then φ is called a *graph isomorphism*. If a graph isomorphism $\varphi : G_1 \to G_2$ exists, then the graphs are called isomorphic and we write $G_1 \cong G_2$.

Let $(V_1, E_1), (V_1, E_1), \ldots, (V_n, E_n)$ be graphs such that $V_i \cap V_j = \varnothing$ for all $i \neq j$. The disjoint union of $(V_1, E_1), (V_1, E_1), \ldots, (V_n, E_n)$ is defined as $\dot{\bigcup}_{i=1}^{n} (V_i, E_i) := \left(\dot{\bigcup}_{i=1}^{n} V_i, \dot{\bigcup}_{i=1}^{n} E_i \right)$. A subgraph F of a graph G is called a *strong subgraph* of G if and only if whenever u and v are vertices of F if u and v are adjacent in G, then u and v are adjacent in F as well.

3. PRINCIPAL IDEAL GRAPHS OF A LEFT GROUPS

We characterize a principal ideal graphs of a left groups in this section. First, we describe the necessary and sufficient conditions for any two elements in a left group S are adjacent in ${}_S L_S$.

Lemma 1. *Let $S = G \times L$ be a left group and $(g_1, l_1), (g_2, l_2) \in S$. Then there exists an edge between (g_1, l_1) and (g_2, l_2) in ${}_S L_S$ if and only if $l_1 = l_2$.*

Proof Let $S = G \times L$ be a left group and (g_1, l_1), $(g_2, l_2) \in S$.

$(\Rightarrow)$ Assume that there exists an edge between (g_1, l_1) and (g_2, l_2) in ${}_S L_S$. Then by the definition of ${}_S L_S$, we thus get $(g_1, l_1)S \cap (g_2, l_2)S \neq \varnothing$. Hence there exists (a, i) and $(b, j) \in S$ such that

$$\begin{aligned} (g_1, l_1)(a, i) &= (g_2, l_2)(b, j) \\ \Rightarrow \quad (g_1 a, l_1) &= (g_2 b, l_2) \\ \Rightarrow \quad l_1 &= l_2 \end{aligned}$$

$(\Leftarrow)$ Let $l_1 = l_2$. Now we have $(g_1, l_1)(g_1^{-1}, i) = (g g_1^{-1}, l_1) = (e, l_1)$, where e is the identity element in G. Also we have, $(g_2, l_2)(g_2^{-1}, i) = (g_2 g_2^{-1}, l_2) = (e, l_2) = (e, l_1)$ as $l_1 = l_2$. This means that, $(g_1, l_1)S \cap (g_2, l_2)S \neq \varnothing$. And now we have $(g_1^{-1}, i)(g_1, l_1) = (e, i) = (g_2^{-1} g_2, i) = (g_2^{-1}, i)(g_2, l_2)$. This means that, $S(g_1, l_1) \cap S(g_2, l_2) \neq \varnothing$. Therefore there exists an edge between (g_1, l_1) and (g_2, l_2) in ${}_S L_S$.

The next theorem, we characterize a principal ideal graphs of a left groups. From now on, p_i denotes the projection into the i^{th} component.

Theorem 2. *A graph (V, E) is a principal ideal graph of a left group if and only if the following conditions holds:*

(1) $(V, E) = \dot{\bigcup}_{i=1}^{m} (V_i, E_i)$ *for some* $m \in \mathbb{N}$;

(2) for all i = 1,2,..,m, (V_i, E_i) *is a complete graph with* t *vertices for some* $t \in \mathbb{N}$.

Proof $(\Rightarrow)$ Let (V, E) be a principal ideal graph of a left group. Then there exists a left group $S = G \times L$ where $G = \{g_1, g_2, \ldots g_t\}$ is a group and $L = \{l_1, l_2, \ldots l_m\}$ is a left zero semigroup, such that $(V, E) \cong {}_S L_S$. Hence we will prove that (1) and (2) are true for ${}_S L_S$.

(1) For each $i \in \{1, 2, \ldots, m\}$, set $V_i = G \times \{l_i\}$, and $E_i = E({}_S L_S) \cap E(K_{|V_i|})$ where $K_{|V_i|}$ is a complete graph with V_i is the vertex set. Hence $S = \dot{\bigcup}_{i=1}^{m} V_i$ and (V_i, E_i) is a strong subgraph of ${}_S L_S$. Let $(g_k, l_i) \in V_i$ and $(g_{k'}, l_{i'}) \in V_{i'}$. By Lemma 1, there is no edge between (g_k, l_i) and $(g_{k'}, l_{i'})$ in ${}_S L_S$ for all $i \neq i'$. Then we thus get (V_i, E_i) and $(V_{i'}, E_{i'})$ are disjoint for all $i \neq i'$. Therefore ${}_S L_S = \dot{\bigcup}_{i=1}^{m} (V_i, E_i)$.

(2) For all i, by the definition of V_i and Lemma 1, we get that there is an edge between any two vertices in (V_i, E_i). This means that (V_i, E_i) is a complete. We have $|V_i| = |G \times \{l_i\}| = |G| \times |\{l_i\}| = |G| = t$. Hence (V_i, E_i) is a complete graph with t vertices for all $i = 1, 2, \ldots, m$.

$(\Leftarrow)$ From (2), we let $V_i = \{v_{i1}, v_{i2}, \ldots, v_{it}\} = \dot{\bigcup}_{k=1}^{t} \{v_{ik}\}$. From (1), we get that $V = \dot{\bigcup}_{i=1}^{m} V_i = \dot{\bigcup}_{i=1}^{m} \dot{\bigcup}_{k=1}^{t} \{v_{ik}\}$. Let $S = G \times L$ be a left group, where $G = \{g_1, g_2, \ldots g_t\}$ is a group and $L = \{l_1, l_2, \ldots l_m\}$ is a left zero semigroup. We will show that $(V, E) \cong {}_S L_S$. Define $f : V \to S$ by $f(v_{ik}) = (g_k, l_i)$. Since $|V| = \left| \dot{\bigcup}_{i=1}^{m} \dot{\bigcup}_{k=1}^{t} \{V_{ik}\} \right| = t \times m = |G| \times |L| = |S|$, it easily seen that f is a bijection from V to S. We will show that f and f^{-1} are homomorphism from (V, E) to ${}_S L_S$.

Let $a, b \in V$, assume that there exists an edge between a and b in (V, E). By (1), we have $a, b \in V_i$ for some $i \in \{1, 2, \ldots, m\}$. Let $a = v_{ik}$ and $b = v_{ik'}$ for some $k, k' \in \{1, 2, \ldots, t\}$. Therefore $f(a) = f(v_{ik}) = (g_k, l_i)$ and $f(b) = f(v_{ik'}) = (g_{k'}, l_i)$. By Lemma 1, there exists an edge between (g_k, l_i) and $(g_{k'}, l_i)$ in ${}_S L_S$. This means that there is an edge between $f(a)$ and $f(b)$ in ${}_S L_S$. Hence f is a homomorphism.

Conversely suppose that there exists an edge between $f(a)$ and $f(b)$ in ${}_S L_S$. By Lemma 1 again, we get that $p_2(f(a)) = p_2(f(b))$. We let $l_i = p_2(f(a)) = p_2(f(b))$ for some $i \in \{1, 2, \ldots, m\}$. This means that $a, b \in V_i$. From (2), there exists an edge between a and b in (V_i, E_i). It follows that there exists an edge between a

and b in (V,E). Therefore f^{-1} is a homomorphism. Then f is a graph isomorphism from (V,E) to ${}_SL_S$. Hence $(V,E) \cong {}_SL_S$. □

By Theorem 2, we have the following theorem.

Theorem 3. *Let $H = \dot{\bigcup}_{i=1}^{m} H_i$ be a finite disjoint union of complete graphs H_i such that for some $t \in \mathbb{N}$, $|V(H_i)| = t$ for all $i = 1,2,\ldots,m$. Then there is a left group S such that ${}_SL_S \cong H$.*

Proof Let $H = \dot{\bigcup}_{i=1}^{m} H_i$ be a finite disjoint union of complete graphs H_i and $|V(H_i)| = t$ for all $i = 1,2,\ldots,m$. And let $S = G \times L$ be a left group with $|G| = t$ and $|L| = m$. By (1) and (2) of Theorem 2, we get that ${}_SL_S = \dot{\bigcup}_{i=1}^{m} (V_i, E_i)$ and for each (V_i, E_i) is a complete respectively. It follows that ${}_SL_S$ is a disjoint union of m complete graphs. By the proof of (1) again, we have $V_i = G \times \{l_i\}$. Hence $|V_i| = |G \times \{l_i\}| =$

$|G| \times |\{l_i\}| = |G| = t$. Therefore ${}_SL_S$ is a disjoint union of m components in which each component is complete with t vertices. Hence ${}_SL_S \cong \dot{\bigcup}_{i=1}^{m} H_i = H$. □

From Theorem 3 we have the following corollary which describe the number of components in the principal ideal graph of a left group.

Corollary 4. *Let $S = G \times L$ be a left group with $|G| = t$ and $|L| = m$. Then the principal ideal graph ${}_SL_S$ is a disconnected graph with m components in which each component is complete with t vertices.*

By Corollary 4, the following corollary is immediate.

Corollary 5. *Let $S = G \times L$ be a left group. Then the principal ideal graph ${}_SL_S$ is a connected graph if and only if $|L| = 1$.*

The following corollary describe the number of edges in the principal ideal graph of a left group.

Corollary 6. *Let $S = G \times L$ be a left group with $|G| = t$ and $|L| = m$. Then the principal ideal graph ${}_SL_S$ has $\frac{t(t-1)m}{2}$ edges.*

Proof Let $S = G \times L$ be a left group *with $|G| = t$ and* $|L| = m$. By Corollary 4, the principal ideal graph ${}_SL_S$ has m components in which each component is complete with t vertices. Therefore each component in ${}_SL_S$ has $\frac{t(t-1)}{2}$ edges. Because there are m components, then the total number of edges in ${}_SL_S$ is $\frac{t(t-1)m}{2}$. □

Example 1. *Let $S = \mathbb{Z}_3 \times L_4$ be a left group, where $\mathbb{Z}_3 = \{\bar{0}, \bar{1}, \bar{2}\}$ and $L_4 = \{l_1, l_2, l_3, l_4\}$.*

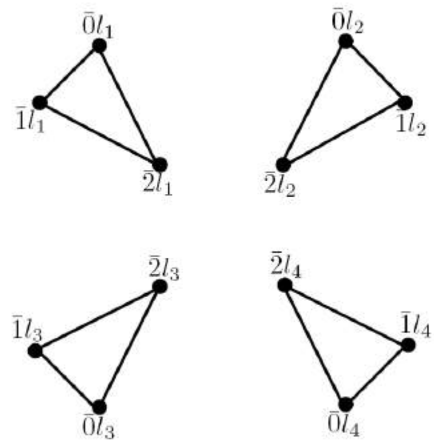

Figure 1 : Principal ideal graph ${}_SL_S$

In Figure 1, we see that ${}_SL_S = \dot{\bigcup}_{i=1}^{4} (V_i, E_i)$ where $V_1 = \{\bar{0}l_1, \bar{1}l_1, \bar{2}l_1\}$, $V_2 = \{\bar{0}l_2, \bar{1}l_2, \bar{2}l_2\}$, $V_3 = \{\bar{0}l_3, \bar{1}l_3, \bar{2}l_3\}$ and $V_4 = \{\bar{0}l_4, \bar{1}l_4, \bar{2}l_4\}$. It is easily seen that ${}_SL_S$ has 4 components $(|L_4 = 4|)$ include that (V_1, E_1), (V_2, E_2), (V_3, E_3) and (V_4, E_4). For each component is a complete with 3 vertices $(|\mathbb{Z}_3 = 3|)$.

4. PRINCIPAL IDEAL GRAPHS OF A RIGHT GROUPS

We characterize a principal ideal graphs of a right groups in this section. First, we describe the necessary and sufficient conditions for any two elements in a right group S are adjacent in ${}_SR_S$.

Lemma 7. *Let $S = G \times R$ be a right group and $(g_1, r_1), (g_2, r_2) \in S$. Then there exists an edge between (g_1, r_1) and (g_2, r_2) in ${}_SR_S$ if and only if $r_1 = r_2$.*

Proof Let $S = G \times R$ be a right group and (g_1, r_1) $(g_2, r_2) \in S$.

$(\Rightarrow)$ Assume that there exists an edge between (g_1, r_1) and (g_2, r_2) in ${}_SR_S$. Then by the definition of ${}_SR_S$, we thus get $S(g_1, r_1) \cap S(g_2, r_2) \neq \varnothing$. Hence there exists (a, i) and $(b, j) \in S$ such that

$$\begin{aligned}(a, i)(g_1, r_1) &= (b, j)(g_2, r_2)\\ \Rightarrow (ag_1, r_1) &= (bg_2, r_2)\\ \Rightarrow r_1 &= r_2\end{aligned}$$

$(\Leftarrow)$ Let $r_1 = r_2$. Now we have $(g_1^{-1}, i)(g_1, r_1) =$

$(gg_1^{-1}, r_1) = (e, r_1)$, where e is the identity element in G. Also we have, $(g_2^{-1}, i)(g_2, r_2) = (g_2^{-1}g_2, r_2) =$

$(e, r_2) = (e, r_1)$ as $r_1 = r_2$. This means that, $(g_1, l_1)S \cap (g_2, l_2)S \neq \varnothing$. And now we have $(g_1^{-1}, i)(g_1, l_1) = (e, i) = (g_2^{-1}g_2, i) = (g_2^{-1}, i)(g_2, l_2)$. This means that, $S(g_1, l_1) \cap S(g_2, l_2) \neq \varnothing$. Therefore there exists an edge between (g_1, l_1) and (g_2, l_2) in ${}_S L_S$. □

The next theorem, we characterize a principal ideal graphs of a right groups.

Theorem 8. *A graph (V,E) is a principal ideal graph of a right group if and only if the following conditions holds:*

(1) $(V,E) = \dot{\bigcup}_{i=1}^{n} (V_i, E_i)$ for some $n \in \mathbb{N}$;

(2) for all i = 1,2,..,n, (V_i, E_i) is a complete graph with t vertices for some $t \in \mathbb{N}$.

Proof $(\Rightarrow)$ Let (V,E) be a principal ideal graph of a right group. Then there exists a right group $S = G \times R$ where $G = \{g_1, g_2, \ldots g_t\}$ is a group and $R = \{r_1, r_2, \ldots r_n\}$ is a right zero semigroup, such that $(V,E) \cong {}_S R_S$. Hence we will prove that (1) and (2) are true for ${}_S R_S$.

(1) For each $i \in \{1,2,\ldots,n\}$, set $V_i = G \times \{r_i\}$, and $E_i = E({}_S R_S) \cap E(K_{|V_i|})$ where $K_{|V_i|}$ is a complete graph with V_i is the vertex set. Hence $S = \dot{\bigcup}_{i=1}^{n} V_i$ and (V_i, E_i) is a strong subgraph of ${}_S R_S$. Let $(g_k, r_i) \in V_i$ and $(g_{k'}, r_{i'}) \in V_{i'}$. By Lemma 7, there is no edge between (g_k, r_i) and $(g_{k'}, r_{i'})$ in ${}_S R_S$ for all $i \neq i'$. Then we thus get (V_i, E_i) and $(V_{i'}, E_{i'})$ are disjoint for all $i \neq i'$. Therefore ${}_S R_S = \dot{\bigcup}_{i=1}^{n} (V_i, E_i)$.

(2) For all i, by the definition of V_i and Lemma 7, we get that there is an edge between any two vertices in (V_i, E_i). This means that (V_i, E_i) is a complete. We have $|V_i| = |G \times \{r_i\}| = |G| \times |\{r_i\}| = |G| = t$. Hence (V_i, E_i) is a complete graph with t vertices for all $i = 1,2,\ldots,n$.

$(\Leftarrow)$ From (2), we let $V_i = \{v_{i1}, v_{i2}, \ldots, v_{it}\} = \dot{\bigcup}_{k=1}^{t} \{v_{ik}\}$. From (1), we get that $V = \dot{\bigcup}_{i=1}^{n} V_i = \dot{\bigcup}_{i=1}^{n} \dot{\bigcup}_{k=1}^{t} \{v_{ik}\}$. Let $S = G \times R$ be a left group, where $G = \{g_1, g_2, \ldots g_t\}$ is a group and $R = \{r_1, r_2, \ldots r_n\}$ is a right zero semigroup. We will show that $(V,E) \cong {}_S R_S$. Define $f: V \to S$ by $f(v_{ik}) = (g_k, r_i)$. Since $|V| = \left| \dot{\bigcup}_{i=1}^{n} \dot{\bigcup}_{k=1}^{t} \{v_{ik}\} \right| = t \times n = |G| \times |R| = |S|$, it easily seen that f is a bijection from V to S. We will show that f and f^{-1} are homomorphism from (V,E) to ${}_S R_S$.

Let $a, b \in V$, assume that there exists an edge between a and b in (V,E). By (1), we have $a, b \in V_i$ for some $i \in \{1,2,\ldots,n\}$. Let $a = v_{ik}$ and $b = v_{ik'}$ for some $k, k' \in \{1,2,\ldots,t\}$. Therefore $f(a) = f(v_{ik}) = (g_k, r_i)$ and $f(b) = f(v_{ik'}) = (g_{k'}, r_i)$. By Lemma 7, there exists an edge between (g_k, r_i) and $(g_{k'}, r_i)$ in ${}_S R_S$. This means that there is an edge between $f(a)$ and $f(b)$ in ${}_S R_S$. Hence f is a homomorphism.

Conversely suppose that there exists an edge between $f(a)$ and $f(b)$ in ${}_S R_S$. By Lemma 7 again, we get that $p_2(f(a)) = p_2(f(b))$. We let $r_i = p_2(f(a)) = p_2(f(b))$ for some $i \in \{1,2,\ldots,n\}$. This means that $a, b \in V_i$. From (2), there exists an edge between a and b in (V_i, E_i). It follows that there exists an edge between a and b in (V,E). Therefore f^{-1} is a homomorphism. Then f is a graph isomorphism from (V,E) to ${}_S R_S$. Hence $(V,E) \cong {}_S R_S$. □

By Theorem 8, we have the following theorem.

Theorem 9. *Let $H = \dot{\bigcup}_{i=1}^{n} H_i$ be a finite disjoint union of complete graphs H_i such that for some $t \in \mathbb{N}$,*

$|V(H_i)| = t$ for all $i = 1,2,\ldots,n$. Then there is a right group S such that ${}_S R_S \cong H$.

Proof Let $H = \dot{\bigcup}_{i=1}^{n} H_i$ be a finite disjoint union of complete graphs H_i and $|V(H_i)| = t$ for all $i = 1,2,\ldots,n$. And let $S = G \times R$ be a right group with $|G| = t$ and $|R| = n$. By (1) and (2) of Theorem 8, we get that ${}_S R_S = \dot{\bigcup}_{i=1}^{n} (V_i, E_i)$ and for each (V_i, E_i) is a complete respectively. It follows that ${}_S R_S$ is a disjoint union of n complete graphs. By the proof of (1) again, we have $V_i = G \times \{l_i\}$. Hence $|V_i| = |G \times \{r_i\}| = |G| = t$. Therefore ${}_S R_S$ is a disjoint union of n components in which each component is complete with t vertices. Hence ${}_S R_S \cong \dot{\bigcup}_{i=1}^{n} H_i = H$.

Similarly to Corollary 4-6 and by using Theorem 9, the three following corollaries are immediate.

Corollary 10. *Let $S = G \times R$ be a right group with $|G| = t$ and $|R| = n$. Then the principal ideal graph ${}_S R_S$ is a disconnected graph with n components in which each component is complete with t vertices.*

Corollary 11. *Let $S = G \times R$ be a right group. Then the principal ideal graph ${}_S R_S$ is a connected graph if and only if $|R| = 1$.*

Corollary 12. *Let $S = G \times R$ be a right group with $|G| = t$ and $|R| = n$. Then the principal ideal graph ${}_S R_S$ has $\frac{t(t-1)n}{2}$ edges.*

ACKNOWLEDGEMENT

The authors would like to thanks the National Research Council of Thailand (NRCT) and Faculty of Science, Nakhon Phanom University.

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