International Journal of Innovative Studies in Sciences and Engineering Technology (IJISSET)

ISSN 2455-4863 (Online) www.ijisset.org Volume: 4 Issue: 5 | May 2018

Mini satellite orbit transfer using Equinoctial elements

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Abstract: *A very sensitive stage in the lifetime of the spacecraft is during the journey from the launch vehicle parking orbit transfer to the final required working orbit. In this paper, altitude orbit transfer method is introduced taking into account the effect of propulsion system operation on the secular classical orbital elements. Elements taken in the study are the semi-major axis (a), eccentricity (e) and the argument of perigee (ω).*

*Hohmann transfer technique is the main method of spacecraft transfer. In this study, an enhancement equinoctial element method is used, which is not strict to execute the maneuver at the apogee or perigee. A case study introduced, it is to transfer a mini satellite with mass 90 kg from a parking orbit 261 * 348 km to a working orbit 315 * 348 km using SSTL-150 Xenon Propulsion System.*

A verification to the orbit transfer method using the Matlab-STK shows an acceptable error of 4% for the used method.

Keywords: *orbit transfer, Equinoctial elements, STK verification.*

1. PROPULSION SYSTEM EFFECT ON SC CLASSICAL ORBITAL ELEMENTS

Orbit transfer scenario includes the time duration of thrusters operation and its exact location per each revolution. This scenario is based mainly on knowing the answer to the question of what is the effect of operating the SC propulsion system (PS) with the specified period of time on the orbital parameters as semi-major axis (a), eccentricity (e) and the argument of perigee (ω).

Pollard in 1997 (Pollard, 1997)stated the change of the orbital parameters with respect to eccentric anomaly, this type of change is secular as expressed in Eq.(1.1) to Eq. (1.3).

$$\frac{da}{dE}=\frac{2a^3}{\mu}\left(f_1 e\sin E+f_2\sqrt{1-e^2}\right) \tag{1.1}$$

$$\frac{de}{dE}=\frac{a^2}{\mu}\left[f_1(1-e^2)\sin E+f_2\sqrt{1-e^2}(2\cos E-e-e\cos^2 E)\right] \tag{1.2}$$

$$\frac{d\omega}{dE}=\frac{a^2}{\mu}\left\{\begin{matrix}\frac{1}{e}\left[-f_1\sqrt{1-e^2}(\cos E-e)+f_2(2-e^2-e\cos E)\sin E\right]\\ -f_3(1-e\cos E)\cot i\left[\frac{(\cos E-e)\sin\omega}{\sqrt{1-e^2}}+\sin E\cos\omega\right]\end{matrix}\right\} \tag{1.3}$$

$$f_2=\frac{Thrust}{m_{SC}} \tag{1.4}$$

Where m_{SC} is the mass of spacecraft (SC) and Thrust is the output thrust force from the propulsion system (PS) thrusters.

Three acceleration components are acting on the SC during its motion as shown in Fig (1.1), f_1 is directed along the radius vector, f_2 is normal to the radius vector on the orbital plane and f_3 is perpendicular to the orbital plane.

The type of orbital maneuvers under study is the altitude change and out of plane maneuvers (inclination correction) are not included, that is why the f_2 acceleration is only used and the other components (f_1 and f_3) are neglected.

The above equations (Eq.(1.1) to Eq. (1.4)) are integrated, simplified and adapted to be used in the complete set of mathematical model of orbital maintenance program.

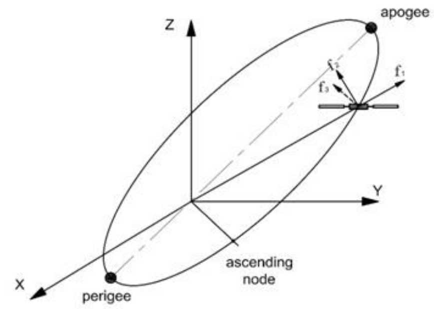

Fig. (1.1) *Acceleration components on spacecraft.*

Secular change of semi-major axis (a) is shown in Eq.(1.5). The equation of eccentricity change for the

International Journal of Innovative Studies in Sciences and Engineering Technology (IJISSET)

ISSN 2455-4863 (Online) www.ijisset.org *Volume: 4 Issue: 5 | May 2018*

continues type of maneuverexecution can be used as the secular change of eccentricity as in Eq.(1.6)and the secular change of the argument of perigee is shown in Eq.(1.7.

$$\Delta a_s = \frac{2a^3}{\mu} f_2\sqrt{1\text{-}e^2}\,\sigma\,(\Delta E) \qquad (1.5)$$

$$\Delta e_s = \frac{a^2}{\mu} f_2\sqrt{1-e^2}\,\{\begin{matrix}[4\sin(E_2)] - [3e(E_2)] \\ -[\frac{e}{2}(\sin 2E_2)]\end{matrix}\} \qquad (1.6)$$

$$\Delta\omega_s = \frac{a^2}{\mu} f_2\,[\begin{matrix}-\frac{2}{e}(\cos E_2 - \cos E_1) - \frac{e}{2}(\sin^2 E_2 - \sin^2 E_1) \\ -\frac{1}{2}(\sin^2 E_2 - \sin^2 E_1)\end{matrix}] \qquad (1.7)$$

The subscript s refers to the secular change of orbital parameters. There is also the long periodic change of the orbital parameters (Dubushin, 1976). This periodic change has only effect on the eccentricity and argument of perigee.

$$\Delta\breve{e} = \frac{3n}{4\dot{\omega}} a_1(1-e^2)\dot{\omega}T_\Omega \sin i \cos\omega \qquad (1.8)$$

$$\Delta\breve{\omega} = -\frac{3n}{4\dot{\omega}} C_1\dot{\omega}\,T_\Omega \sin\omega + \dot{\omega}T_\Omega \qquad (1.9)$$

$$a_1 = \frac{1}{2}\gamma_3(4-S^2) - \frac{20}{32}(8-28S^2+21S^4) + \frac{280}{2048}(64-432S^2+792S^4 - 429S^6) \qquad (1.10)$$

$$C_1 = -S\left[\frac{1}{2}\gamma_3(4-5S^2) - \frac{20}{32}\gamma_5(8-28S^2+21S^4) + \right.$$

2802048γ764−432S2+792S4−429S6 (1.11)

$$\gamma_k = -J_k\,(\frac{a}{p})^k \qquad (1.12)$$

$$S = \sin(i) \qquad (1.13)$$

Where, the constants a_1 and C_1 are constants of long periodic change (Dubushin, 1976). The overall change of the orbital parameters is the summation of secular and periodic variation.

$$\Delta a_{man} = \Delta a_s \qquad (1.14)$$

$$\Delta e_{man} = \Delta e_s + \Delta\breve{e} \qquad (1.15)$$

$$\Delta\omega_{man} = \Delta\omega_s + \Delta\breve{\omega} \qquad (1.16)$$

The set of equations (Eq.(1.1) to Eq.(1.16)) can now be implemented into a simulation program to obtain different responses of orbital parameters with time.

2. EQUINOCTIAL ELEMENTS METHOD

The above-mentioned orbit transfer technique is built to change the SC from its elliptical parking to working orbit with normal equinoctial parameters change, there is a modified technique to decrease the number of maneuvers used and hence decrease the propellant budget and the overall orbit transfer duration.

The new technique is proposed by a Russian scientist in 2007 (Kolegov, 2007) named Kolegov, and its result is approved later in the Russian launched spacecrafts by the Soyuz-series launch vehicle.

To reach a working orbit with an elliptical shape and not circular (0< e <1) while maintaining the argument of perigee, in the normal orbit transfer method, the change in equinoctial parameters must pass first to the origin (0,0), point O on Fig. (2.1), followed by maneuvers, which will increase the eccentricity while maintaining the change in argument of perigee (ω) as small as possible (maneuvers should be carried as fast as possible with the maximum available thrusting force). The new enhancement to the orbit transfer model works on going directly to the required eccentricity without passing to the circular orbit with origin (going directly to point O′).

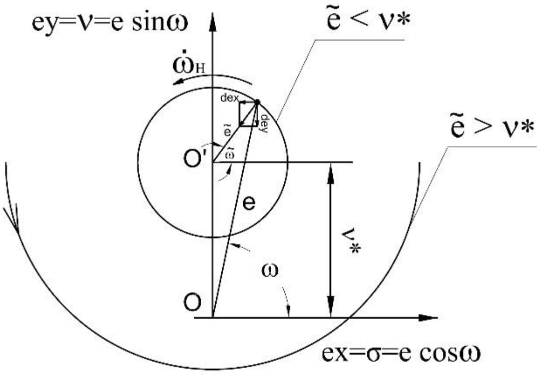

Fig. (2.1) *Eccentricity and argument of perigee transformation.*

The equinoctial elements e_x and e_y are expressed in Eq. (2.1) and Eq. (2.2).

$$e_x = \sigma = \tilde{e}\cos\widetilde{\omega} \qquad (2.1)$$

$$e_y = \nu = \tilde{e}\sin\widetilde{\omega} + \nu^* \qquad (2.2)$$

$$\widetilde{\omega} = \widetilde{\omega}_o + \dot{\omega}_H(t-t_o) \qquad (2.3)$$

The arbitrary variables$\tilde{e}$and$\widetilde{\omega}_o$are determined from the initial equinoctial elements σ_o and ν_o. From Fig. (2.1) the variables $\tilde{e}$ and $\widetilde{\omega}_o$ can be written as follows:

$$\tilde{e} = \sqrt{\sigma_o^2 + (\upsilon_o - \upsilon^*)^2} \qquad (2.4)$$

$$\widetilde{\omega}_o = \tan^{-1}\frac{\upsilon_o - \upsilon^*}{\sigma_o} \qquad (2.5)$$

If the vector with magnitude e uniformly rotates around the point O at a rate $\dot{\omega}_H$, corresponding vector with magnitude $\tilde{e}$rotates around the point O'with the same angular velocity $\dot{\omega}_H$,Fig. (2.1). Using Fig. (2.1) and equation (2.1), the elements e andω can be calculated by:

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ISSN 2455-4863 (Online) www.ijisset.org Volume: 4 Issue: 5 | May 2018

$$e = \sqrt{\tilde{e}^2 + 2\tilde{e}\upsilon^* \sin\tilde{\omega} + \upsilon^{*2}} \quad (2.6)$$

$$\omega = \tan^{-1}\frac{\tilde{e}\sin\tilde{\omega} + \upsilon^*}{\tilde{e}\cos\tilde{\omega}} \quad (2.7)$$

The value of the ν* corresponds to the required final eccentricity, and by knowing the values of (e) and (ω), the values of $\tilde{e}$ and $\tilde{\omega}_0$ can be calculated. To reach the required eccentricity denoted by ν* in Fig. (2), the change of (e) and (ω) must follow the line denoted by ($\tilde{e}$) as in Eq. (2.8).

$$\frac{dey}{dex} = \tan\tilde{\omega} \quad (2.8)$$

Change in the equinoctial elements are expressed in Eq. (2.9) and Eq. (2.10).

$$dex = \sqrt{\frac{a}{\mu}}\, 2\cos(u)\,\Delta V \quad (2.9)$$

$$dey = \sqrt{\frac{a}{\mu}}\, 2\sin(u)\,\Delta V \quad (2.10)$$

Dividing Eq. (2.10) with Eq. (2.9) we obtain.

$$\frac{dey}{dex} = \tan(u) \quad (2.11)$$

From Eq. (2.8) and Eq. (2.11), change of argument of perigee ($\tilde{\omega}$) can be directly related to the argument of latitude (u) as in Eq. (2.12).

$$\tan(u) = \tan(\tilde{\omega}) \quad (2.12)$$

From Eq. (2.12), the location of the center point of maneuver execution (u) is related directly to the change of the argument of perigee ($\tilde{\omega}$).

3. PROPULSION SYSTEM SELECTION

Empirical data for the propulsion system mass selection is a fraction of the total mass of SC. Percentage of propulsion system from the total SC mass is from 10 ÷ 15 %, which guides the mass selection of propulsion system in the range 9 ÷ 15 kg.

Another information is important for the propulsion system selection is the total propellant budget required to transfer the SC to an orbit with a lifetime of one year. A model is built on STK with the same SC data used for lifetime calculation; altitude is increased until the lifetime reaches one year. Increasing the orbit perigee from 261 km to an altitude 315 km will increase the lifetime of SC to exactly 1 year, this change in orbit will make the final operational orbit in an elliptical shape 315 * 348 km.

Using the normal Hohmann transfer from the elliptical orbit, a transfer from an initial elliptical orbit 261 * 348 km to a final elliptical orbit 315 *348 km requires a velocity budget 15.5 m/s. Finally, the required propulsion main constrains is as follows:

- Total mass in the range 10 ÷ 15 kg approximately.

Total velocity budget generated at least 15.5 m/s.There are fully integrated propulsion systems that sold commercially, the leading British company named SURREY Satellite Technology LTD (SSTL) works in space and specialized mainly in the production of small satellites; it is a leading and most cooperative company with many other countries in the field of Research and Development. The company also offers different types of propulsion systems; the most suitable is the SSTL-150 (as in Fig. (3.1)). The SSTL-150 Xenon Propulsion System (LTD, 2014) consists of two separate items, the propellant storage and supply assembly and the thruster assembly. The propellant feed system including the controller electronics is assembled on a single honeycomb panel. The propellant is fed to the single thruster using a series of redundant solenoid valves, which are used in a bang-bang control mode(on–off controller).The single thruster, when used with SC pitch control, provides a wide range of maneuvering capabilities.

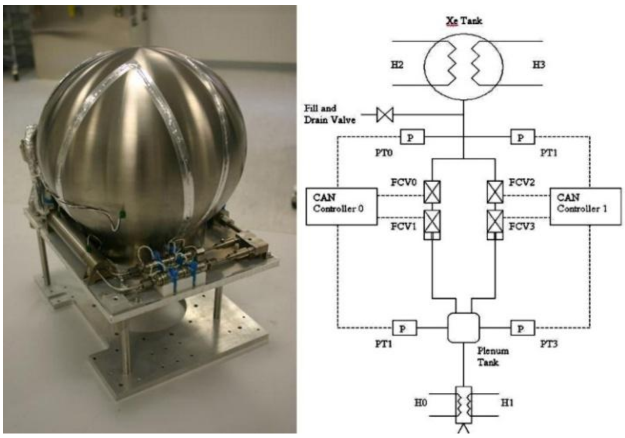

Fig.(3.1) *SSTL-150 Xenon Propulsion System* (LTD, 2014).

The system can be described as a warm gas system. It uses electrical power to heat a resistojet thruster and increase the specific impulse of the xenon propellant over that gained using it in a cold gas mode. The system can be used to provide launcher injection correction, station keeping and acquisition, orbit height maintenance and de-orbit maneuvers resistojet thrusters and propulsion gas tank system are designed & configured for easy integration. The datasheet of system is shown in Table (3.1).

International Journal of Innovative Studies in Sciences and Engineering Technology (IJISSET)
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Table (3.1): *SSTL-150 Xenon prop. System specification* (LTD, 2014)

Parameter	***Value***
Propellant mass at BOL	12.00 kg of Xenon
Dry mass	7.3kg
Nominal thrust	18 mN
Thruster heater power	2 * 30 W (prime and redundant)
Nominal warm up period prior to firing	700 seconds
Thruster Isp at 500°C	48 seconds nominal
Total nominal impulse	5644 Ns
Delta V on **a 385 kg platform**	14.9 m/s
Thermal	System to be maintained at > 17º C to avoid liquid phase xenon
Electrical interface	28Vdc power and CAN bus
Connectors	44 way D type connector
Thruster thermal qualification	3441 thermal cycles from 50°C to 500°C
Propellant tank	7.42 Litre Spherical Pressure Vessel Ti6Al 4V Burst pressure > 480 Bar MEOP = 120 Bar

Where, MEOP is the maximum Expected Operational Pressure.

Provided data of the SSTL-150 propulsion system shows a total mass (12 kg propellant + 7.3 kg dry mass) 19.3 kg. The total produced Delta V for such system is 14.9 m/s. Provided total Delta V is less than the required to perform orbit transfer, but it is stated that it is for a SC with a mass 385 kg. Orbit transfer and the SSTL-150 propulsion system data is used by Hohmann transfer equations and the required propellant mass to perform a transfer for a SC with 90 kg mass is 2.94 kg.

Required mass of propellant is less than the available mass in the system, that is why another tank with smaller capacity can be chosen, and the dry mass will be lowered and the propellant mass. For simplicity, it is assumed that the excessive propellant mass will only be removed from tank and the net mass of system will be 10.8 kg (3.5 kg is filled to tanks).

4. ORBIT TRANSFER TO WORKING ORBIT

Initial Hohmann transfer equations (Capderou, 2005)approved the capability of the SSTL-150 propulsion system to transfer the 90 kg SC from its parking orbit 261 * 348 km to the required one-year operational orbit 315 * 348 km. According to the orbit transfer model using the equinoctial parameters, parking orbit 261 * 348 kmis used as an initial input data to the MATLAB based transfer model. Orbit transfer model yields an output of total number of maneuvers (134), maneuvers duration pattern is shown in Fig. (4.1)). Maneuvers duration are in the average of 9.5 min with a total duration 1301.7 min, at which the thrusters are put in operation. However the SC rotates around the Earth for about 14.3 revolution per day. Therefore, the whole orbit transfer would take about 9.5 days.

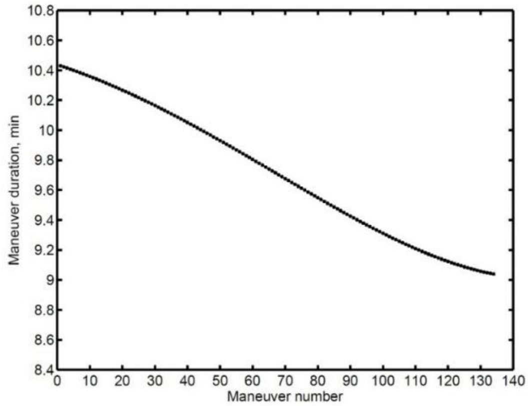

Fig. (4.1) *Maneuver duration for the SC orbit transfer.*

Location of maneuvers center is also indicated for all maneuvers according to the orbit transfer enhancement. All maneuvers are executed around the Apogee and it is a normal expected result; as the Perigee is only required to be raised in altitude. Fig. (4.2) shows the start and end true anomaly location of maneuver, the center of maneuvers starts at 180° (Apogee) and increases slightly for every successive maneuver until the last maneuver is executed at a true anomaly 197°.

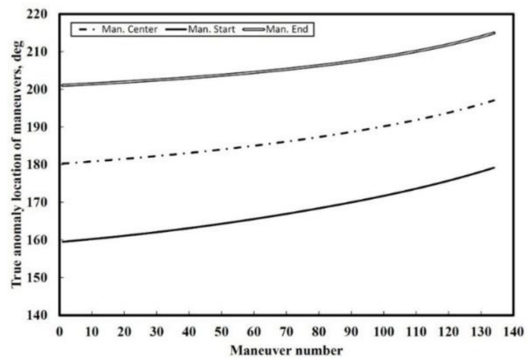

Fig. (4.2) *True anomaly location of each maneuver.*

Change in the classical orbit elements are focused mainly on the perigee altitude (or final reached altitude) and the eccentricity change with argument of perigee, which is the equinoctial elements change represented by e_x and e_y as shown in Fig. (4.3)). The upper right point is the start point of maneuvers propagation, it shows the argument of perigee starts at

92.876° as the separation (ω). The shape of the equinoctial elements change is of elliptical shape, the elliptical shape expresses active propagation of SC motion under effect of maneuvers, as the circular shape is for the passive motion. Equinoctial change at the end of maneuvers shows a final (ω) 168.13°, final reached ex is -0.002031 and ey 0.0004269 with a final eccentricity 2.1E-3. The change from the initial parking orbit (261 * 348 km) to the (315 * 348 km) operational orbit gives the change of eccentricity from 6.509E-3 to 2.459E-3 using eccentricity relation. The model change of the eccentricity shows a change from the initial value 6.509E-3 to a final reached eccentricity 2.1E-3, Fig(4.4).

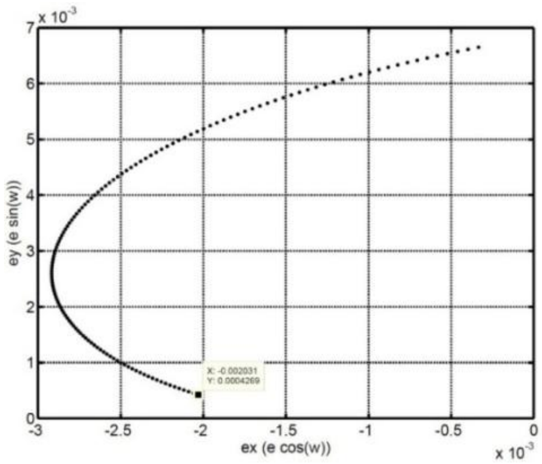

Fig. (4.3) *Equinoctial parameters variation for orbit transfer maneuvers.*

Perigee altitude is the final required output from model, Fig. (4.5) shows perigee altitude variation under the effect of each corresponding maneuver. Altitude change shows an average increase 44.6 m for every one minute of maneuver; for a total maneuvers duration 1301 min, perigee altitude is raised 58 km from 261 km to 319 km.

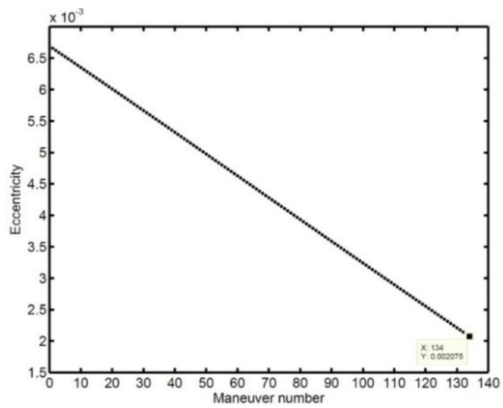

Fig.(4.4) *Eccentricity change over orbit transfer maneuvers.*

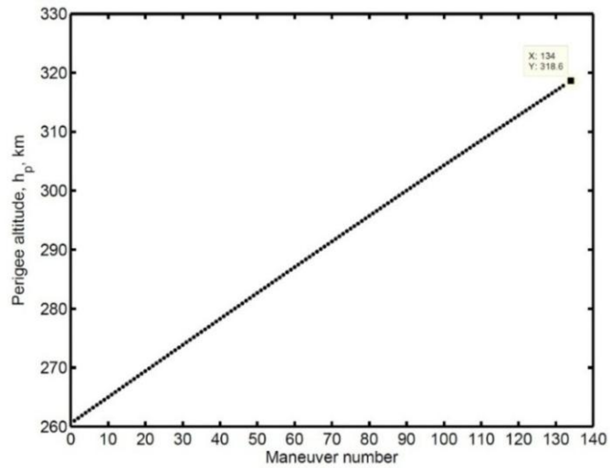

Fig.(4.5) *Perigee altitude under effect of corresponding maneuver.*

5. ORBIT TRANSFER VERIFICATION

To reach a higher confidence level of the designed model before the use of designed data on the actual SC, designed data is propagated on a simulation module of the STK. STK package includes the capability to be connected with the MATLAB, and by the use of MATLAB scripts integrated in STK, the SC mass, mass moment of inertia, used SC propulsion system as well as the time of execution of maneuvers are fed to the tailored scenario in STK. STK-MATLAB scripts provides the STK with the starting and end times of all the designed maneuvers. The SC propagates during the passive part of revolution by the STK propagation engine, while during the designed maneuvers part, SC moves actively in space under the effect of input thrusters data from the MATLAB. By the end of all designed maneuvers, a report can be generated to analyze the final orbit reached, where it can be compared to the required working orbit.

Maneuvers location and duration are added to the scripts of MATLAB-STK model (Mathworks, 2012). Total transfer duration for the (134) maneuvers is about (9.5) days, orbit transfer starts with the parking orbit 261 * 348 km and the final required operational orbit is 315 * 348 km. STK model with active maneuvers is propagated, all the maneuvers are executed approximately over the same location. Location of maneuvers is over the southern part of South America continent and all around the Apogee location as in Fig(5.1).

Final reached orbit from propagation is 317 * 348 km, required raise of perigee is 54 km, and the model shows a total raise 56 km. The deviation of the model for this transfer is 4%.

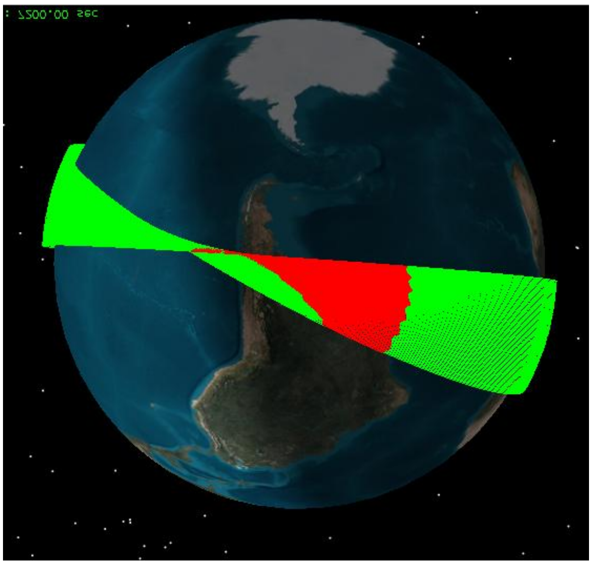

Fig. (5.1) *Active maneuvers location during orbit transfer.*

6. CONCLUSION

The effect of spacecraft propulsion system operation on the change on the classical orbital elements such as semi-major axis (a), eccentricity (e) and the argument of perigee (ω) are introduced. Also the equinoctial method that indicates the location of execution of maneuver is also described. A mathematical model for the orbit transfer including the effect of PS and the maneuver location is deduced and implemented on a Matlab platform.

A case study of a mini satellite of mass 90 kg simulating the transfer of its orbit from the parking orbit 261 * 348 km to the working orbit 315 * 348 km is introduced. Total number of maneuvers required, equinoctial elements variation, perigee altitude and the eccentricity variation during the orbit transfer are shown and verified using the Matlab-STK interface. An acceptable deviation of 4% of the whole orbit transfer mission is indicated.

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