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Common Fixed Point in Fuzzy Metric Space

Gauri Shanker Sao

Department of Mathematics, Govt. Err Pg Science College Bilaspur(C.G.), India

Abstract:

The aim of present paper is to obtain a fixed point by contraction in fuzzy metric space. Our purpose is to generalize the several known results.

Keywords and Phrases: fuzzy 2-metric spaces, contraction mapping, compability, fixed point.

AMS subject classification(2000): 54H25, 47H10.

1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [14] in 1965, who introduced the concept of fuzzy set. George and Veeramani [3] modified the notion of fuzzy metric spaces by introducing the concept of continuous t – norm. The fuzzy metric space was introduced by Kramosil and Michalek [5],Khan M.S, Pathak H.K. & George R.[6] compatible mappings of Type (A−1) and Type A−2 in fuzzy metric spaces, Sao,G.S. and Shrivastava,U.K. [11] introduce the notion of compability of type(D).

2. PRELIMINARIES

The following definitions and results will be needed in the sequel.

DEFINITION 2.1([13]) Let X be any set. A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition(2.2)[3]: Abinary operation: $[0,1]\text{x}[0,1]\rightarrow[0,1]$ is a continuous t-norm if $\{[0,1],*\}$ is an abelian topological monoid with unit 1 such that $a*b\leq c*d$ whenever $a\leq c$ and $b\leq d$, $a,b,c,d\in[0,1]$

Definition(2.3)[3]:A **fuzzy metric space** is a space X in which for each triplet of points x,y there exits a real function M(x,y,t) such that :

(i)to each pair of distinct points x,y in X there exists a point t in[0,1] s.t. $M(x,y,t)\neq 0$;

(ii) $M(x,y,t) = 1$, when x,y are equal;

(iii)$M(x,y,t)= M(y,x,t)$;

(iv)$M(x,y,t_1+t_2)\geq M(x,y,t_1) * M(x,y,t_2)$

for all x,y in X and t_1,t_2 in [0,1].It is easily seen that M is non -negative.

Definition(2.4)[3]:A sequence$\{x_n\}$in a fuzzy metric space(X,M,$*$)is said to be **convergent** with lim x in X if

$$\lim_{n\to\infty} M(x_n,x,t)=1$$

for all x,y inX and t>0.

Definition(2.5)[3]:A sequence $\{x_n\}$ in a fuzzy metric space (X,M,$*$) is said to be a **Cauchy sequence** if

$$\lim_{m,n\to\infty} M(x_m,x_n,t)=1$$

for all x,y in X and t>0.

Definition(2.5)[3]:A fuzzy metric space (X,M,$*$)is said to be **complete** fuzzy **metric space** if every Cauchy sequence in X is convergent.

DEFINITION 2.6([1]) Two self-mappings A and S of a fuzzy metric space (X,M,$*$) are said to be compatible if,

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence such that:

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.7([4]) Two self-mappings A and S of a fuzzy metric space (X,M,$*$) are said to be weakly compatible if, they commute at coincidence points.That is,Ax=

Sx implies that $ASx=SAx$ for all x in X.

DEFINITION 2.8([2]) Two self-mappings A and S of a fuzzy metric space (X,M,$*$) are said to be subcompatible if, $\lim_{n\to\infty} M(ASx_n, SAx_n,t) = 1$ whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.9([7]) Two self-mappings A and S of a fuzzy metric space (X,M,$*$) are said to be reciprocally continuous if,

$$\lim_{n\to\infty} M(ASx_n, Ax, t) = 1$$

and $\lim_{n\to\infty} M(SAx_n, Sx, t) = 1$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

It is noted that if A and S are both continuous, they are obviously reciprocally continuous but the converse need not be true. For this, we have the following example:

DEFINITION 2.10([6]) Two self-mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be compability of type(A) if,

$$\lim_{n\to\infty} M(ASx_n, SSx_n, t) = 1$$

and $\lim_{n\to\infty} M(SAx_n, AAx_n, t) = 1$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

3. MATERIAL AND METHOD

Theorem(3.1):Let (X,M,*) be a fuzzy metric space and let $T,S:X\to X$ are continuous mappings of X, then $\{Tx_n\}$and $\{Sy_n\}$ converges to p.If T and S satisfying the following condition:

$M(Tx,TSy,t)\leq\min(M(Tx,Ty,t),$

$M(y.TSy,t),M(Ty,STx,t)\}$

When x,y in X and $t\in[0,1]$,then T and S has a common fixed point p.

4. RESULT AND DISCUSSION

Proof of theorem:Consider

$Q_n = M(y_n,y_{n+1},t)$if $y_n=Tx_n = x_{n+1}=Sy_n$ then we have

$Q_n\leq\min\{Q_n,Q_n,Q_n\}$

So Q_n is a Cauchy fuzzy metric space.. Also we can easily prove that it is convergent.

$\Rightarrow M(Tx_n,p,t)\to 1$

Therefore it converge at the point p .

Also we can easily show that point p is unique.

Therefore T and S have a common fixed point p .

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