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Fixed Point in Hilbert Spaces

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Abstract: *In the present paper,we establish some fixed point results for contractive type Hilbert spaces.*

Keywords and Phrases: *Hilbert space, Contraction, Cauchy sequence.*

AMS Subject classification: *Primary 47H10,Secondary 54H25*

1. INTRODUCTION

In recent years some fixed points of various type of compability mapping in Hilbert space and Banach spaces were obtained by Browder [1], Browder and Petryshyn[2], Hicks and Huffman[3], Jungck[4].

2. PRELIMINARIES

2.1 **NORM** : A norm on X is a real-valued function $||.|| : X \to R$ defined on X such that for any x, y $\in$ X and for all $\lambda \in K$

(a) $||x|| = 0$ if and only if $x = 0$

(b) $||x+y|| \le ||x|| + ||y||$

(c) $||\lambda x|| = |\lambda|\, ||x||$

2.2 **NORMED LINEAR SPACE** : It is a pair (X, ||.||) consisting of a linear space X and a norm ||.||. We shall abbreviate normed linear space as nls.

2.3 CAUCHY SEQUENCE : A Sequence $\{x_n\}$ in a normed linear space X is a Cauchy sequence if for any given $\varepsilon > 0$, there exist $n_0 \in N$ such that $||x_m - x_n|| < \varepsilon$ for $m, n \ge n_0$

2.4 **CONVERGENCE CONDITION IN NLS** : A sequence $\{x_n\}$ in a nls X is said to be Convergent to $x \in X$ if for any given $\varepsilon > 0, \exists$ $n_0 \in N$ such that $||x_n - x|| < \varepsilon$ for $n \ge n_0$

2.5 **COMPLETENESS** : A nls X is said to be complete if for every Cauchy Sequence in X converges to an element of X.

2.6 **BANACH SPACE** : A Banach Space (X, ||.||) is a complete nls.

2.7 **INNER PRODUCT SPACE** : Let X be a linear space over the scalar field C of complex numbers. An inner product on X is a function $(. , .) : XxX \to C$ which satisfies the following conditions

(a) $(x, y) = \overline{(y, x)}$ for x, y $\in$ X

(b) $(\lambda x + \mu y, z) = \lambda (x, z) + \mu (y, z)$ for $\lambda, \mu \in C$, x, y, z $\in$ X

(c) $(x, x) \ge 0$; $x\ x) = 0$ iff $x = 0$

2.8 **LAW OF PARALLELOGRAM:** If x and y are any two elements of an inner product space X then $||x + y||^2 + ||x-y||^2 = 2||x||^2 + 2||y||^2$

2.9 **HILBERT SPACE** : An infinite dimensional inner product space which is complete for the norm induced by the inner product is called Hilbert Space.

3. MATERIAL AND METHOD

Theorem:Let C be a closed subset of a Hilbert space H. $T:C \to C$ are contraction and continuous map of C , then T converges to p. If

$$\|Tx-Ty\|^2 \le a_1\|x-Tx\|^2 + a_2\|y-Ty\|^2 + a_3\|x-Ty\|^2 + a_4\|y-Tx\|^2 + a_5\|x-y\|^2 + a_6\frac{\|x-Tx\|^2(1+\|x-Tx\|^2)}{1+\|x-y\|^2}$$

then T converge to unique fixed point, when $a_1 + a_2 + 4a_3 + a_4 + a_5 + a_6 < 1$

4. RESULT AND DISCUSSION

Proof of theorem: It can be proof in easy ways Consider

$Tx_0 = x_1$ and $Tx_1 = x_2$

Let $Tx_n = x_{n+1}$

then $Tx_{n+1}=x_{n+2}$

when substituting $x=x_n$ and $y=x_{n+1}$ then by (i)

$$\|Tx_n - Tx_{n+1}\|^2 \le a_1\|x_n - Tx_n\|^2$$

$$+a_2\|x_{n+1} - Tx_{n+1}\|^2 + a_3\|x_n - Tx_{n+1}\|^2$$

$$a_4\|x_{n+1} - Tx_{n+1}\|^2 + a_5\|x_n - x_{n+1}\|^2$$

$$+ a_6 \frac{\|x_n - Tx_n\|^2(1+\|x_n - Tx_n\|^2)}{1+\|x_n - x_{n+1}\|^2}$$

$$\Rightarrow \|x_{n+1} - x_{n+2}\|^2 \le a_1\|x_n - x_{n+1}\|^2$$

$$+a_2\|x_{n+1} - x_{n+2}\|^2 + a_3\|x_n - x_{n+2}\|^2$$

$$a_4\|x_{n+1} - x_{n+2}\|^2 + a_5\|x_n - x_{n+1}\|^2$$

$$+ a_6 \frac{\|x_n - x_{n+1}\|^2(1+\|x_n - x_{n+1}\|^2)}{1+\|x_n - x_{n+1}\|^2}$$

$$\Rightarrow \|x_{n+1} - x_{n+2}\|^2 \le a_1\|x_n - x_{n+1}\|^2$$

$$+a_2\|x_{n+1} - x_{n+2}\|^2 + 2a_3\|x_n - x_{n+1}\|^2$$

$$+2a_3\|x_{n+1} - x_{n+2}\|^2 + a_4\|x_{n+1} - x_{n+2}\|^2$$

$$+a_5\|x_n - x_{n+1}\|^2$$

$$+ a_6 \frac{\|x_n - x_{n+1}\|^2(1+\|x_n - x_{n+1}\|^2)}{1+\|x_n - x_{n+1}\|^2}$$

by law of parallelogram

$$\Rightarrow \|x_{n+1} - x_{n+2}\|^2 (1-a_2-2a_3-a_4)$$

$$\le (a_1+2a_3+a_5+a_6)\|x_n - x_{n+1}\|^2$$

$$\Rightarrow \|x_{n+1} - x_{n+2}\|^2 \le k\|x_n - x_{n+1}\|^2$$

Where $k=\dfrac{a_1+2a_3+a_5+a_6}{1-a_2-2a_3-a_4} < 1$

$\Rightarrow a_1+a_2+4a_3+a_4+a_5+a_6<1$

$\Rightarrow$ $\{x_n\}$ is Cauchy sequence , then it is easily claim that $\{x_n\}$ is convergent by the properties of contraction .

5. CONCLUSION

Therefore $Tx_n=p$

Hence p is unique fixed point of T

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