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Common Fixed Point in Fuzzy Metric Space of Integral Type

G.S. Sao, A.K.Sao, Seema Sinha & U.K.Shrivastava

Department of Maths, Govt. ERR PG. Sc. College, Bilaspur (C.G.), India

Abstract: *Fuzzy metric space is first defined by Kramosil and Michalek in 1975. Many authors modified Fuzzy metric space and proved fixed point results in Fuzzy metric space. Singh B. and Chauhan were first introduced the concept of compatible mappings of Fuzzy metric space and proved the common fixed point theorem in 2000.Cho et were introduced the concept of compatible mapping of type (P). In this paper, we obtain a fixed point in fuzzy metric space .Our purpose is generalize the several known results.*

Keywords: *Common fixed points, fuzzy metric space.*

2000 AMS Subject Classification: *Primary 47H10, Secondary 54H25*

1. INTRODUCTION

Rhoades (1985) proved for pair of mappings which in turn was generalized by Kang and Rhoades (1996) using compatible condition defined by Jungck (1986). Branciari (2002) obtained a fixed point result for a single mapping satisfying an analogue of Banach's contraction principle for an intregral-type inequality. The second auther (Rhoades, 2003) prove Two fixed point theorems involving more general contractive conditions.

2. PRELIMINARIES

tisfies Consider $\Phi = \{\varphi : \varphi : \Re^+ \to \Re\}$ such that φ is nonnegative, Lebesgue integrable, and sa

$$\int_0^{\varepsilon} \varphi(t)dt > 0 \text{ for each } \varepsilon > 0 \qquad (2.1)$$

Let $\psi : \Re^+ \to \Re^+$ satisfy that

(i) ψ is nonnegative and nondecreasing on $\Re^+$,

(ii) $\psi(t) < t$ for each $t > 0$,

(iii) $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each fixed t>0. where

$\Psi = \{\psi : \psi$ satisfies (i)-(iii)}.

3. MATERIAL AND METHODS

Theorem (3.1):Let A, B, S and T be the mappings from a fuzzy metric space (X,M,*) into itself satisfying the following conditions:

(i) S and T are surjective

(ii) One of A, B, S and T is continuous

(iii) A, S and B, T are compatible pairs of type (P)

(iv) $$\int_0^{M(Ax,By,z)} \varphi(t)dt \le \psi\left(\int_0^{m(x,y,z)} \varphi(t)dt\right) \quad (3.1)$$

Where $k \in [0,1], \varphi \in \Phi, \psi \in \Psi$,and

m(x,y,z)=max{M(Sx,Ty,z),M(Sx,Ax,z),M(Ty,By,z),

$$\frac{\max\{M(Sx,Ty,z).M(Sx,By,z), M(Sx,Ty,z).M(Ty,Ax,z), M(Sx,Ax,z).M(Sx,By,z), M(Ty,By,z).M(Ty,Ax,z)}{\max\{M(Sx,By,z), M(Ty,Ax,z)\}}$$

} (3.2)

when m(x,y,z)=1 if Sx=By and Ty=Ax.Then A,B,S and T have a unique common fixed point in X.

4. RESULTS AND DISCUSSION

Proof of theorem: If A,B,S and T be the mappings from a fuzzy metric space (X,M,*)into itself then there exists a sequence $\{x_n\} \subset X$ with $x_0 \in X$,

$Ax_{2n} = Tx_{2n+1} = x_{2n+1}$ and $Bx_{2n+1} = Sx_{2n+2} = x_{2n+2}$

Now assume $x_{2n} \ne x_{2n+1}$ for each n. With $x=x_{2n}, y=x_{2n+1}$,then from (3.1) we have

$$\int_0^{M(Ax_{2n},Bx_{2n+1},z)} \varphi(t)dt \le \psi\left(\int_0^{M(x_{2n},x_{2n+1},z)} \varphi(t)dt\right)$$

$$\Rightarrow \int_0^{M(x_{2n+1},x_{2n+2},z)} \varphi(t)dt \le \psi\left(\int_0^{M(x_{2n},x_{2n+1},z)} \varphi(t)dt\right) \quad (3.3)$$

Continuing this process, we have

$$\int_0^{M(Ax_{2n},Bx_{2n+1},z)} \varphi(t)dt \le \psi\left(\int_0^{M(x_{2n},x_{2n+1},z)} \varphi(t)dt\right) \le \ldots \le \psi^{2n}(M) \quad (3.4)$$

where $M = \int_0^{M(x_0,x_1,z)} \varphi(t)dt$. Then it is easily shown that $\{x_n\}$ is Cauchy, hence convergent. Call the limit p. Consequently the subsequences$\{Ax_{2n}\}$,$\{Bx_{2n+1}\}$, $\{Sx_{2n}\}$, $\{Tx_{2n+1}\}$ converge to p .

Let p=Sp=Ap .Then m(p,p,z)= max {M(Sp,Tp,z),M(Sp,Ap,z),M(Tp,Bp,z),

$$\frac{\max\{M(Sp,Tp,z).M(Sp,Bp,z), M(Sp,Tp,z).M(Tp,Ap,z), M(Sp,Ap,z).M(Sp,Bp,z), M(Tp,Bp,z).M(Tp,Ap,z)}{\max\{M(Sp,Bp,z), M(Tp,Ap,z)\}}$$

} (3.5)

Therefore m(p,p,z)= max {M(p,Tp,z),1,M(Tp,Bp,z),

$$\frac{\max\{M(p,Tp,z).M(p,Bp,z), M(p,Tp,z).M(Tp,p,z), 1.M(p,Bp,z), M(Tp,Bp,z).M(Tp,p,z)}{\max\{M(p,Bp,z), M(Tp,p,z)\}}$$

} (3.6)

Hence M(p,Tp,z).M(p,Bp,z)=1 since M(Tp,Bp,z) $\leq$ M(Tp,p,z)*M(p,Bp,z)

So p=Tp, p=Bp and (3.1) becomes

$$\int_0^{M(p,Tp,z)} \varphi(t)\,dt \leq \psi\left(\int_0^{M(p,Tp,z)} \varphi(t)\,dt\right) \qquad (3.7)$$

which, from (3.3), implies that p=Tp=Bp.

Similarly, p=Tp=Bp implies that p=Sp=Ap. We will now show that A, B, S and T satisfy (3.5).

m(x,Sx,z)=maxM(Sx,TSx,z),M(Sx,Ax,z),M(TSx,BSx,z)

$$\frac{\max\{M(Sx,TSx,z).M(Sx,BSx,z), M(Sx,TSx,z).M(TSx,Ax,z), M(Sx,Ax,z).M(Sx,BSx,z), M(TSx,BSx,z).M(TSx,Ax,z)}{\max\{M(Sx,BSx,z), M(TSx,Ax,z)\}}$$

(3.8).

Now we have M(Sx,TSx,z) $\leq$ M(Sx,Ax,z)*M(Ax,TSx,z)

M(Sx,BSx,z) $\leq$ M(Sx,Ax,z)*M(Ax,BSx,z)

M(BSx,TSx,z) $\leq$ M(BSx,Bx,z)*M(Bx,TSx,z)

on the considering above we have

$\int_0^{M(Sx,TSx,z)} \varphi(t)dt \leq \psi \left(\int_0^{M(Sx,TSx,z)} \varphi(t)dt\right.$ a contradiction

5. CONCLUSION

Hence S,A and T,B has a fixed point ,that any fixed point of S,A is also a fixed point of T,B and conversely. Thus S,A and T,B have a common fixed point . Hence A,B,S and T has a common fixed point.

Suppose that T is continous. Since B and T are compatible, then TTx_{2n+1} =Tp and BBx_{2n+1}=Tp.

So that M(TTx_{2n+1}, BBx_{2n+1}, z)=1=M(SSx_{2n},AAx_{2n} ,z)=1 as $n \to \infty$

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