

COMPABILITY OF TYPE (A) IN HILBERT SPACES

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Abstract: In the present paper, we prove compability of type (A) through contraction mapping in Hilbert spaces.

Keywords and Phrases: Hilbert space, Contraction, compability, Cauchy sequence.

AMS Subject classification: Primary 47H10, Secondary 54H25

1. INTRODUCTION

In recent years some fixed points of various type of compability mapping in Hilbert space and Banach spaces were obtained by Browder [1], Browder and Petryshyn [2], Hicks and Huffman[3], Jungck [4].

2. PRELIMINARIES

2.1 NORM : A norm on X is a real-valued function $\|\cdot\| : X \rightarrow \mathbb{R}$ defined on X such that for any $x, y \in X$ and for all $\lambda \in \mathbb{K}$

(a) $\|x\| = 0$ if and only if $x = 0$

(b) $\|x+y\| \leq \|x\| + \|y\|$

(c) $\|\lambda x\| = |\lambda| \|x\|$

2.2 NORMED LINEAR SPACE : It is a pair $(X, \|\cdot\|)$ consisting of a linear space X and a norm $\|\cdot\|$. We shall abbreviate normed linear space as nls.

2.3 CAUCHY SEQUENCE : A Sequence $\{x_n\}$ in a normed linear space X is a Cauchy sequence if for any given $\epsilon > 0$, there exist $n_0 \in \mathbb{N}$ such that $\|x_m - x_n\| < \epsilon$ for $m, n \geq n_0$

2.4 CONVERGENCE CONDITION IN NLS : A sequence $\{x_n\}$ in a nls X is said to be Convergent to $x \in X$ if for any given $\epsilon > 0$, $\exists n_0 \in \mathbb{N}$ such that $\|x_n - x\| < \epsilon$ for $n \geq n_0$

2.5 COMPLETENESS: A nls X is said to be complete if for every Cauchy Sequence in X converges to an element of X.

2.6 BANACH SPACE: A Banach Space $(X, \|\cdot\|)$ is a complete nls.

2.7 INNER PRODUCT SPACE : Let X be a linear space over the scalar field C of complex numbers. An inner

product on X is a function $(\cdot, \cdot) : X \times X \rightarrow \mathbb{C}$ which satisfies the following conditions

(a) $(x, y) = \overline{(y, x)}$ for $x, y \in X$

(b) $(\lambda x + \mu y, z) = \lambda (x, z) + \mu (y, z)$ for $\lambda, \mu \in \mathbb{C}, x, y, z \in X$

(c) $(x, x) \geq 0 ; (x, x) = 0$ iff $x = 0$

2.8 LAW OF PARALLELOGRAM: If x and y are any two elements of an inner product space X then $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$

2.9 HILBERT SPACE: An infinite dimensional inner product space which is complete for the norm induced by the inner product is called Hilbert Space.

2.10 s-biased of type(A):

$$\alpha \|Sx_n - Ax_n\|^2 \leq \alpha \|ASx_n - Sx_n\|^2$$

where $\alpha = \liminf x_n = \limsup x_n$

3. MATERIAL AND METHOD

Theorem: Let C be a closed subset of a Hilbert space H. $A, S: C \rightarrow C$ are contraction and continuous map of C, then AS converges to p. If

$$\|ASx - SAy\|^2 \leq a_1 \|ASy - y\|^2 + a_2 \|ASx - y\|^2 + a_3 \|Ax - Sy\|^2 \quad (i)$$

then A and S have a unique common fixed point

4. RESULT AND DISCUSSION

Proof of theorem: It can be proof in easy ways Consider

$$Ax_0 = x_1 \quad \text{and} \quad Sx_1 = x_2$$

Let $Ax_n = x_{n+1}$

$$Sx_{n+1} = x_{n+2}$$

when substituting $x = x_n$ and $y = x_{n+1}$ then by (i)

$$\|ASx_n - SAx_{n+1}\|^2 \leq a_1 \|ASx_{n+1} - x_{n+1}\|^2$$

$$\begin{aligned}
 &+a_2 \|ASx_n - x_{n+1}\|^2 + a_3 \|Ax_n - Sx_{n+1}\|^2 \\
 \Rightarrow &\|x_{n+2} - x_{n+3}\|^2 \leq a_1 \|x_{n+3} - x_{n+1}\|^2 \\
 &+a_2 \|x_{n+2} - x_{n+1}\|^2 + a_3 \|x_{n+1} - x_{n+2}\|^2 \\
 \Rightarrow &\|x_{n+2} - x_{n+3}\|^2 \leq 2a_1 \|x_{n+3} - x_{n+2}\|^2 + \\
 &2a_1 \|x_{n+2} - x_{n+1}\|^2 + a_2 \|x_{n+2} - x_{n+1}\|^2 \\
 &+a_3 \|x_{n+1} - x_{n+2}\|^2 \text{ by law of parallelogram} \\
 \Rightarrow &\|x_{n+2} - x_{n+3}\|^2 (1-2a_1) \\
 \leq &(2a_1 + a_2 + a_3) \|x_{n+1} - x_{n+2}\|^2 \\
 \Rightarrow &\|x_{n+2} - x_{n+3}\|^2 \leq k \|x_{n+1} - x_{n+2}\|^2
 \end{aligned}$$

Where $k = \frac{2a_1 + a_2 + a_3}{1 - 2a_1} < 1$

$\Rightarrow \{x_n\}$ is Cauchy sequence, as well as compability of type(A) then it is easily claim that $\{x_n\}$ is convergent by the properties of compability of type(A), also by the properties of s-biased of type(A) it can be easily prove that s-biased of type(A).

5. CONCLUSION

Therefore $Ax_n = p = Sx_n$

Hence p is unique fixed point of A and S

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