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Battery Testing with the Calculated Discharge Curve Method - Real and Virtual Discharge

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Abstract: *The Calculated Discharge Curve Method is based on thermodynamically reversible work: the product of the open-circuit voltage, initial current, and time, i.e., the sum of useful energy and energy losses. A calculated discharge curve is based on the constant step change of the battery voltage in correspondence with a cardinal number set. The essential solution is the transformation of the discharge data voltage vs. time into time vs. voltage using basic equations (three-point operators of power of internal resistance, time intervals and subintervals, capacities and energies), which are valid for all battery electrochemical systems, cell/mono-block/ battery designs and discharge conditions. The mono and multi-cell battery operating conditions are: (1) the four discharge modes (constant loads: resistor, current, voltage and power); (2) two load regimes: self-driving and device-driving (galvanostat, potentiostat) or battery con-nection (serial, parallel, combine); and (3) continual and intermittent discharge. The battery average cell, average characteristics (regarding time and capacities) and suc-cessive and corresponding ratios, cross-ratios and cross-ratio equations are introduced as the new battery cha-racteristics. The all of these characteristics and inter-relations are concerned to real and virtual, derived from real, discharges and are performed for real cell/mono-block/battery and virtual cells, i.e. the energy parts of cell's overall energy volume.*

1. CDCM CELL/BATTERY TESTING

This Paper is based on more than one published papers, and summary are described in publication: *Battery tes-ting with the calculated discharge curve method - 3D Mathematical Model, LAP Lambert Academic Publishing (2015-12-24) ISBN: 978-3-659-80353-6, in further cita-tion: 'CDCM'.*

The objective characterisation of the C/B (B denotes multi-cell battery, of any connections of single cells) is to determine discharge parameters for optimal and reliable power supply. The fundamental parameters are voltage and current, their capacities and powers on load and energy losses, the all of which are acting during a discharge time. CDCM - 3D Mathematical Model is proposed as the new CDCM procedure which is in accordance to Standard battery electrochemistry, but introduces the new, very useful improvements. CDCM may be applied to any electrochemical system, C/B electrochemical model, cell design and cells' connections (serial and/or parallel). CDCM is using the same input data registered as for Standard procedure, i.e. 'voltage vs. time'. These data, 'voltage vs. time', CDCM is transforming into 'time vs. voltage'.

CDCM introduces the new C/B presentation and characterization:

a) C/B overall energy parallelepiped: voltage- current parallelogram times with time altitude,

b) double discharge curve: time interval and time subintervals, as altitude(s) of overall energy parallelepiped and subintervals layers respectively,

c) 2P(points)-operators, i.e. average characteristics, capacities vs. time, and energies vs. capacities,

d) 3P(points)-operators which unities the three subsequent values of C/B electrochemical parameters,

e) virtual discharges from real one: constant resistive-current-voltage-power,

f) virtual cells inside a real/vessel one: voltage-current - electrochemical overvoltage losses, and

g) virtual constant power, electrochemically reversible, discharge identical to the both (one and two) calorimeter readings.

Analysing electrochemical systems, C/B design, load regimes and intensities CDCM introduces:

- self-driving - constant resistor, device-driving (constant current - voltage - power) as well as battery's (instead device) reversible and irreversible discharges,
- intensive and extensive parameters, real and virtual discharges and
- real (vessel, container) and virtual C/B cells as the parts of vessel's cell.

The mathematical calculations were processed on PC Microsoft 'Excel' software. There are enough published C/B discharge data for the CDCM evaluation to actual stage. But, for the sequence:

real discharge > CDCM recalculated virtual discharge > performed virtual discharge

there is no published data. In this paper the set of C/B self-driving discharges are proposed in accordance to CDCM procedure for solving the interrelations between self-driving and device-driving discharge parameters by the aim to be enable to analyse multi-cell battery.

Author chooses Electrochemical Thermodynamic calorimetry tests as the approach to CDCM procedure starting with:

Thermodynamic Overall Energy Volume

Recalling the definitions from 'Electrochemical Methods [Allen J. bard, Larry R Faulkner]' about reversibility, CDCM acquainted and accepted the next decisions:

- *Chemical Reversibility: "...Reversing the cell current merely reverses the cell reaction. No*

new reactions appear, and thus the cell is termed chemically reversible...".

- *Thermodynamic Reversibility: "...A process is thermodynamically reversible when an*

infinitesimal reversal in a driving force causes the process to reverse its direction..."

- ***Practical reversibility:*** *"...Since all actual processes occur at practical rates, they cannot proceed with strict thermodynamic reversibility. However, they may in practice be carried out in such a manner that thermodynamic equations apply to any desired accuracy..."*

Basic Electrochemical Thermodynamic *confirm practical reversibility experimentally:*

- With the one calorimeter reading:

"...During the discharge, heat will evolve from the resistor and from the cell, and we could measure the total heat change by placing the entire apparatus inside a calorimeter. We would find that the heat evolved is 233 kJ/mol of Zn, independent on R. That is ΔH^o = 233kJ, regardless of the rate of cell discharge..." and

- With the two calorimeter readings:

"...Let us now repeat the experiment with the cell and the resistor in separate calorimeters. Assume that the wires connected them have no resistance and do not conduct any heat between the calorimeters. If we take Q_c as the heat absorbed by the cell and Q_R as that dissipated in the resistor, we find that $-Q_c + Q_R$ = 233 kJ/mol of Zn reacted, independent on R. However, the balance between these quantities does depend on the rate of discharge. As R increases $|Q_c|$ decreases and $|Q_R|$ increases..."

CDCM is using mono-cell LR20 VARTA, real, continual, self-driving discharge data, 10 Ohm (1.2), to recover interrelation between Electrochemical Thermodynamic and CDCM results. One calorimeter reading, containing cell and resistor, is equal to the sum of the two calorimeters readings: resistor-useful electrical work and cell's energy losses. CDCM overall energy volume in the form of parallelepiped with base (open voltage times with initial current) is equal to calorimeter readings recalculated thermodinamical into electrochemical units.

These energies has been accepted as *electrochemicaly reversible discharges* and are putted in CDCM procedure*:*

- to transform calorimeter's reading into the electrochemical discharge energy parameters,
- to generate 3D-discharge figure: Overall Energy Volume defined with: initial voltage and current as base (FIGURE 1) and discharge time as time altitude (FIGURE 2), equal to calorimeter(s) reading(s) and C/B nominal capacity,
- to include C/B irreversible discharge parameters inside Overall Energy Volume, obtaining CDCM 3D-geometry figure (FIGURE 3) of the relevant discharge parameters the all relevant intensive and extensive properties.

CDCM offers Laboratory Task Program: C/B discharges of the same electrochemical system and C/B design, under the four discharge modes and the various intensities to realize the researche sequence:

real discharge > virtual discharge > realized virtual
tester author tester

TABLE: *Real and virtual discharge*

Parameter	Resistor	Current	Voltage	Power	Battery response
Resistor*	*Real*	Virtual	Virtual	Virtual	Voltage/current vs time
Current	Virtual	*Real*	Virtual	Virtual	Voltage vs time
Voltage	Virtual	Virtual	*Real*	Virtual	Current vs time
Power	Virtual	Virtual	Virtual	*Real***	Time

This experimental sequence is the crucial CDCM innovation to battery electrochemistry. The all discharges presented in *TABLE: Real and virtual discharge* are used in standard battery testing, but CDCM is asking that interrelation between the modes be defined. The final result will be the differences between self and device driving discharges!

The three resistors are proposed in Labaratory Task, i.e. the three self-driving discharge. These discharges are CDCM input data for the three virtual discharges recalculated with CDCM Algorithm. And finally, the recalculated parameters are input data for real device-driving discharge..

CDCM OVERALL ENERGY VOLUME WITH THE ONE CALORIMETER READING

CDCM accepts Basic Electrochemical Thermodynamic practical reversibility with the one calorimeter reading, defining Overall Energy Volume (OEV) with electrochemically parameters: open-circuit voltage, constant resistor load and time duration. OEV is in the form of right parallelepiped i.e. the product: *Initial voltage x Initial current x Final discharge time* which is visible in Figure 3, excluding the irreversible time altitudes inside parallelepiped [1,2]. Initial voltage and initial current define OEV's parallelepiped base: PP which is covered with net defined by constant voltage and current steps, Figure 1,. There are 'N' volta-ge steps (open-circuit voltage divided with constant voltage-step) as well as 'N' current steps (voltage step divided with constant resistor value) which form N^2 elementary parallelograms. CDCM introduces the cardi-nal number set: 0 < i <= N by which the corres-pondences between the CDCM generated time and time dependent C/B parameters are defined. The first parallelogram is positioned at UoIo point. The all other positions are defined with pairs: voltage and current. Positioned time interval at parallelogram, ordered triple: voltage-cur-rent-time interval is formed, defining energy para-lleledpiped in correspondence to 0 < i <= N set.

Divided adiabatic calorimeter energy reading (OEV) with PP (initial voltage x initial current) final discharge time is calculated. The N^2 final time alti-tudes, i.e. time intervals, define N^2 elementary para-llelepipeds which sum is equal to OEV. Electroche-mically and by CDCM, this OEV is *virtual constant power reversible C/B discharge*.

OEV parallelepiped is bounded with: bottom and top power plane (PP) and the two pairs: voltage-time and current - time, lateral faces. On the PP, CDCM procedure generates N^2 parallelograms, i.e. N^2 time interval parallelepipeds. Every time interval parallelepiped contains N CDCM time elementary subintervals:

$$'\Delta t\ minimum < \Delta t\ maximum > \Delta t\ minimum'$$

Summary, CDCM recognized: N^2 parallelograms, N^2 time interval parallelepipeds and N^3 CDCM time sub-interval parallelepipeds as C/B OEV electrochemical pa-rameters. Every energy parallelepiped is defined with: bottom and cover power base, and voltage-current-time points, generating curved lines- surfaces-volumes.

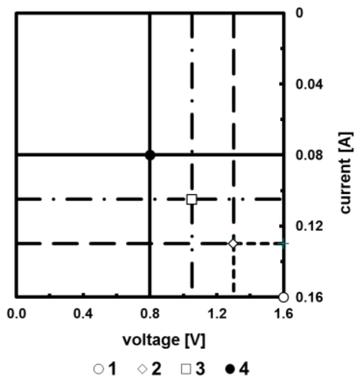

Figure 1: *CDCM battery power plane, Mono-cell battery LR20 VARTA,*

1. Initial power, UoIo/VA 2. Actual power on load, (VI)i/VA

3. Actual average power, (VI)avg,i/VA 4. Final power on load, (VI)n/VA

If one calorimeter C/B discharge is registered by battery tester, there are 'voltage vs. time discharge curve. CDCM task is to recalculate 'voltage vs. time' into CDCM 'time vs. voltage', i.e. time-steps set: time vs. voltage, for the constant voltage and current step, defining the time subintervals set, which sum is equal to final time interval, satisfying sequence:

$$'\Delta t_{initial} < \Delta t_{maximum} > \Delta t_{final}'$$

Starting from the initial (UoIo $\Delta t_{initial}$) OEV's energy layer, every next layer is defined with the new time subintervals. C/B Constant power virtual reversible discharge is presented with N energy layers of the various time subintervals in accordance to ordered triple: voltage-current-time subinterval.

If this numerical sets are positioned inside 3D OEV, the 3D CDCM discharge curve is generated.

CDCM generates 3D energy space for the any electrochemical system, battery design, discharge mo-des, discharge regimes, and load intensities. On this way CDCM was putted intensive (voltage, current, po-wer) and extensive (voltage and current capacities and energies) C/B discharge parameters in 3D space in the corresponded interrelation including indexing cardinal number set: $0 < i < n = N/2$, see Figure 3.

CDCM divides the thermodynamically reversible OEV's volume into electrochemically recognized parts: energy on load and CDCM losses: a) voltage-current losses, b) voltage losses-current and c) voltage losses-current losses.

Initial CDCM PP, i.e. the base of OEV' paralle-lepiped is electrochemical defined regarding C/B proce-sses, i.e. OEV known structure: energy on load and energy losses. Whatever is C/B discharge electro-che-mistry the initial PP remains unchanged during virtual thermodynamic reversible discharge giving final time as CDCM electrochemical parameter in accordan-ce to 3D geometry:

voltage x current x time

and named OEV's parallelepipeds. Any deviation from virtual reversibility leads to real OEV's inside structure which CDCM procedure is enable to analysed .

2. CDCM DISCHARGE TIME GENERATION TIME INTERVAL AND TIME SUBINTERVALS

After defining OEV PP in the form of a parallelogram, OEV in the fom of parallelepiped may be cal-culated with either:

- **first:** defining energy parallelepipeds with: power parallelograms and discharge time interval,

- **second:** defining elementary energy parallelepipeds with: power parallelograms and discharge time

subintervals,

- **third:** as the sum of step energy layers defined with the electrochemical parallelogram base. OEV comprises:

N^2 energy parallelepipeds of N^2 elementary parallelograms: $\Delta V\Delta I t_{interval}$ (VAs).

A Δt layer comprises N_2 elementary energy parallelepipeds: $\Delta V\Delta I\Delta t_{subinterval}$ (VAs), and OEV comrisses N_2 Δt layers.

N^3 elementary energy parallelepipeds of N^2 elementary parallelograms: $\Delta V\Delta I\Delta t_{subinterval}$ (VAs).

The most important part of CDCM research, in terms of defined electrochemical characterization, was started from time definitions:

Time: "A limited stretch or space of continued existence, as the interval between two successive events or acts, or the period through which an action, condition or state continues."

Space: "Lapse or extent of time between two definite points, events, etc.

To define interrelation between 'voltage vs. time' and 'time vs.voltage' CDCM has been studying:

- the discharge duration, $t_{interval}$ = f(initial and dischrge voltage and current) and

- the voltage step change duration $\Delta t_{subinterval}$ = f(initial and dischrge voltage and current),

concluding that the any OEV's parts with known bases, depends on the both, time interval and time sub-interval. CDCM accepts that: time interval interrelated with previous discharge steps, while actual time step interrelated with concrete voltage step. Every actual time subinterval, Δt/s defines: layer's energy, voltage and current capacities, as well as actual time interval, $t_{interval}$/s defines: C/B discharged energy, voltage and current capacities.

CDCM recognized the fact that discharge time is the sum of unequal subintervals and accepts the citation [Numerical Methods, Terence J. Akay, p. 181]:

"The fundamental curve-fitting problem in two dimensions is to predict from a discrete set of (x, y) data pairs (x_1, y_1) through (x_n, y_n) the value of y when the value of x is specified. The least–square approximation of Chapter 4 is one form of curve fitting in which we seek the parameters of a given model that best fit the data".

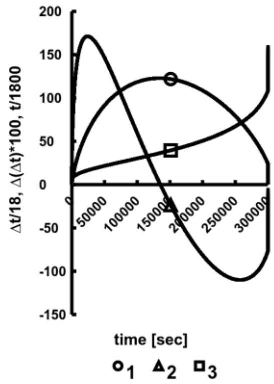

Figure 2: *Time-steps and average time vs. time interval*

1. Time-step, Δti/18,

2. The steps of Time-step, Δ(Δti)/100,

3. Average time, $t_{avg,i} = t^{-1\,1it}$

In CDCM approach we look at other methods which do not looking for random errors of the x,y values. We

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instead, treat the experimental data as if they were accurate and determine a curve that passes through the data points as close as possible. CDCM derives basic equation using the set of po-wer of internal resistance, 0 < Pint,i <= Pint,n, which is time independent and comprises electro-chemically relevant values: overvoltage and actual current. CDCM parameters are the both, nu-merical (0 < i < n = N/2, ΔV, Δη, ΔI, Δ**ν**) and electrochemical parameters (V, I, t, Δt). These parameters and there algebraic relations are in the mutual correspondence.

In the presented LR20 VARTA cell every set contains n = N/2 = 320 members which cover the discharge vol-tage interval: Uo > Vdischarge >= Vfinal. Self-driving and continual discharge of mono-cell battery, ac-ross 10 Ohm, was registered by HP acquisition unit. CDCM input data is the set 'voltage vs. time' obtained from of C/B discharge data in Standard procedure. The twelve (12) triple: i-th position – voltage/V – time/s is extrac-ted from the registered set, HP 3054 DL computer and were used in CDCM procedure (LAMBERT-book, Figure 3). The only one of 12 time values has the role of the common point of input and generated time sets, as the initial 3P-time operator member. Battery tester frequ-ency reading is very high, and the registered set con-tains high number of pairs 'voltage vs time'. CDCM extract the pairs which voltage value is enough close to the values predefined in CDCM voltage set. The number of these pairs need to trace 'S' shape of registered dis-charge curve. CDCM generates the all pairs of CDCM set using only one pair from Input set and estimated the subsequent pair, either before or after the selected initial. Estimation leads to generate CDCM set which satisfy the real discharge curve.

Defining OEV's with generated discharge time intervals and time subintervals, CDCM analyses their interrela-tions regarding: discharge mode, useful and loosed energies, voltage and current capacities, subsequent and corresponded parameters, direct and averaged C/B parameters, load intensity, and any C/B responses. The algebraic result is presented in Figure 2. and with the next relations*:*

$$\mathbf{i\Delta t_i = t_i + \Sigma i\Delta(\Delta t)_i} \qquad \text{(a)}$$

$$\mathbf{it_i = \Sigma t_i + \Sigma i(\Delta t)_i} \qquad \text{(b)}$$

$$\mathbf{i^2\Delta t_i = it_i + i\Sigma i\Delta(\Delta t)_i} \qquad \text{(c)}$$

$$\mathbf{i^2 t_i = i\Sigma t_i + i\Sigma i(\Delta t)_i} \qquad \text{(d)}$$

$$\mathbf{i\Delta S_{Vi} = S_{Vi} + \Sigma i\Delta(\Delta E_V)_i} \qquad \text{(e)}$$

$$\mathbf{i\Delta E_{VIi} = E_{VIi} + \Sigma i\Delta(\Delta E_{VI})_i} \qquad \text{(f)}$$

The electrochemical meanings of these equations are obtained by multiplying equations with constant vol-tage/current steps (ΔV, Δη, ΔI, Δν), as well as by their summation, to define capacity surfaces and energy volumes. Further analyses of these relations are not addressed in this Proposal.

The generated time basic equation is based on the 3P(points) operator which comprises the three succes-sive values of time (or time-dependent parameters) that is equal to 3P(points) operator of power of internal resistance. The ratio of two successive 3P-time opera-tors is equal to two the ratio of two successive 3P(points) power operators of internal resistance. At the end of the generated time estimation to experimen-tal data, the time cross-ratio is equal to internal power cross-ratio. Since the two successive time values are known the third will be defined. The basic equation generates the discharge time set, which overlaps the experimental time values and is accepted by CDCM for characterizing the battery during its exploitation as a standard discharge curve. The discharge time is discharge duration, i.e. time intervals and energy layers durations, i.e. time subintervals. In consequence, CDCA discharge time is presented by the time intervals and time subintervals as the two sets in parallel. This fact is CDCM improve to cell/battery testing and is pre-scented in this repot. CDCM has studied of the discharge time algebraic relations.

1. CDCM VIRTUAL REVERSIBLE OVERALL ENERGY VOLUME

OEV extent and structure, for defined PP, depend on the time interval and the time subintervals. The time subintervals define N energy layers. In the Standard procedure 'voltage vs. time' every of N sub-intervals is of the same time-step, and layer's energy depends on voltage/current. In the CDCM 'time vs. voltage' procedure, Δ(VI) = constant, while time interval and subinterval are changed, defining for every of N elementary layer, i.e. as well as for every of N^2 elementary parallelepiped inside layer.

The elementary parallelepipeds inside a time subinterval layer are of the same energy contain, i.e. of the same bases and time subintervals. Also, the all N^3 elementary parallelepipeds inside OEV, has the same base parallelograms. The energy layers' time subintervals are changed in accordanse to the set: $\Delta t_{minimum} < \Delta t_{maximum} > \Delta t_{minimum}$.

PP, i.e. the bottom base of LR20 VARTA self-driving, constant resistor, continual discharge is shown in Figure 1. The overall (initial voltage and cu-rent) parallelogram is marked with point 1. The next three are: actual power on load - point 2, final average power - point 3 and final power on load - point 4. These parallelograms are surfaces at zero time and are the projection from corresponded parallelogram at defined time, see Figure 3. The final parallelograms of energy on load is filled with N^2/4 elementary parallelograms and marked the top of 'hill' of energy on load, visible in Figure 3. The average power is the both bottom and top bases (constant power) of right parallelepiped (generated by CDCM as 2P(oints) operator) and is of the equal energy contain as the actual 'energy on load in the form of 'hill'. The all denoted parallelograms may be used for either for virtual or real discharge (constant power segments of PP), and the both are equal to resistor calorimeter reading.

Generally, CDCM defines the segments of PP (as the assemblage of elementary parallelograms) in accordance to C/B electrochemical model regarding to C/B electrochemical system, C/B design, dis-charge mode and load intensity.

These segments of PP suggest to CDCM to introduce virtual cell as the part of OEV inside the designed cell's vessel as real cell. In Figure 1 CDCM generates the next virtual cells: (1) on load, (2) voltage losses, (3) current and (4) CDCM losses. Virtual cells have no 'vessel', but CDCM procedure treats these cells via their electrochemical parameters: voltage and current interval with CDCM discharge time.

Over the every parallelogram on PP (defined with pair: voltage vs current) the corresponded 3D dis-charge time curve time may be placed, which is visible in Figure 3. It is interesting to describe the smallest virtual cell 'ΔVΔItactual', i.e. one of N^2 OEV elementary energy parallelepiped. This energy space is defined with: position on PP, containing the both energy on load and energy losses, their lateral faces, i.e. defined voltage and current capacities. The defined number of these smallest parallelepipeds filled the real as well as virtual cells. A PP may be defined and used with either electrochemical voltage and current or CDCM average voltage and current, applying in constant power the both virtual and real discharge (battery tester).

CDCM has published data concerned to multi-cell battery in serial connection were represented with battery average cell. The use of average cells of either serial or parallel battery parts is the familiar way for definition PP of multi-cell battery.

2. CDCM OVERALL ENERGY VOLUME WITH THE TWO CALORIMETERS' READINGS

In addition to adiabatic calorimetry reading CDCM continues to analyse OEV's using two calorimeters. PP and OEV are the same as is that for one calorimeter, Figure 1. CDCM investigates the difference between energy on load (resistor in the first calorimeter) and energy losses (battery in the second calorimeter). Be-fore calorimeter readings, CDCM defines these energies with the next equations:

$$OEV = U_{initial} I_{Iinitial}\, t_{final} = [(Vt+\eta t)(It+\nu t)]t_{final}$$

$$OEV_t = [VI+V\nu+\eta I+\eta\nu]t_{final}$$

where: VI *=on load and* Vν+ηI+ην= *eneregy losses*

where '$U_{initial} I_{initial}$' is electrochemically known PP and time final is calculated with OEV (sum of calorimeter readings) divided with OEV base, i.e. product of initial voltage and initial current. OEV base is composed of N^2 elementary parallelepipeds which need to be positioned in accordance to equation(s) above, i.e. inside one of the four PP's segments. These segments are visible on the both Figure 1 and Figure 3. As well as in OEV with one calorimeter the constant power, continual electrochemical virtual discharge is performed with N^2 time internal parallelepipeds (of the same number of the smallest parallelograms) defined with the final discharge time. So, the empty OEV paralleled, obtained with one calorimeter reading, is transformed into the OEV parallelepiped filled with the N^2 virtual parallelepipeds of the same final discharge time (as the constant power discharge mode) and grouped these in the four PP segments:

Resistor calorimeter: - *energy on load*

C/B calorimeter: - *energy losses: a. current, b. voltage and c. CDCM.*

Every single virtual parallelepipeds is filled either with parts: energy on load and energy losses. Electrochemically, as well as with one calorimeter, OEV is also, energy space of a virtual reversible device driving constant power discharge. The N^2 elementary parallelepipeds of the final time altitude are divided into the four energy segments. CDCM accepts these segments as virtual sells, i.e. C/B vessel is the real CDCM cell, while the four energy spaces inside real C/B are *CDCM virtual cells*. Every CDCM virtual cell undergoing to the same analysing as well as the real cell.

In Figure 1, the four points denotes the four parallelograms: 1. Initial power, UoIo/VA, 2. Actual power on load, (VI)i/VA, 3. Actual average power, (VI)avg,i/VA and 4. Final power on lo-ad, (VI)n/VA. Excluded actual average voltage the all others parallelograms and their altitudes are visible in Figure 3.

The term 'virtual' (introduced in CDCM procedure) denotes cell and/or discharge which calculable with real parameters may be performed, but may not be performed in real experiment. Constant discharge PP's parameters in the virtual generated OEV are corresponded to real ones: *selfdriving resistor, constant current, voltage or power*.

Otherwise, the constant power reversible discharge (PP parallelograms between the bottom and the top bases remain unchanged), is calculable, is not performed but is possible to be, *is virtual discharge*. CDCM is enabling to generate virtual discharge which is corresponded to real discharge, as well as to analyse their interrelations regarding to the common parameters: time - voltage - current -capacities - energies.

The next equations quantified this experiment introducing C/B power base. $U_{initialIinitial}$, Figure1 across the next procedure with CDCM averaged parameters:

$$E_{OEV}= U_{initialIinitial}t_{final} =$$
$$=[(V_{average}+\eta_{average})(I_{average}+\nu_{average})]t_{final}$$

$$OEV_t= [(VI)_{average}+(V\nu)_{average}+(\eta I)_{average}+(\eta\nu)_{average}]\ t_{final}$$

When this equation is analysed to the details, CDCM is using any time subinterval as time step either from 'voltage vs. time' or CDCM from 'time vs. voltage'. PP became very operative representing C/B discharges regarding to: effective electrochemical sys-tem, C/B de-sign, cells connection, load modes and intensities, pro-ducing the corresponded time depen-dent intensive (averaged voltages, currents, powers) and extensive voltage and current capacities and ener-gies. Finally, looking at any CDCM PP one can recognize at least the two virtual cells: on load and on energy lo-sses, and at the same time, the four virtual cells: on load, voltage and current losses and CDCM, as the parts of OEV vessel cell. These four virtual cells are visible in Figure 1 and Figure 3.

Summary: *C/B discharge by one and two calorimeters, CDCM recalculates in electrochemical discharges (re-sistor, current, voltage, power) and contitue investiga-tion with 'time vs. voltage' input data. Real and virtual cells and discharges may be interrelated. Every discharge mode genereted from real one is characterized with own PP!*

3. THE POSSIBLE FORMS OF DISCHARGE TIME GENERATION

The logarithmic form of Eq. (a) comprises electro-chemical power of internal resistance, while in Eq. (b) the polynomial ratio is used for defining 3P- time ope-rators. The same dependences are used in Eq. (c) and Eq. (d). In Eq. (e) instead 'Δt, t' the 't, Σt' is used which is in accordance to interrelation between time steps and time intervals.

$$F[\ln(t)] = \ln[t_i t_i-1]\ln[[t_i-1_{ti-2}]] =$$
$$\ln[P_{int,i}P_{int,i-1}]\ln[[Pint_{i-1}Pint_{i-2}]] = G[\ln(Pint)] \quad (a)$$
$$F[\ln(t)]=\ln[t_i t_{i-1}]\ln[ti-1t_{i-2}] = G[polinomial] \quad (b)$$
$$F[\Delta t,t] = \Delta t_i t_i-2\Delta t_i-1t_{i-1}= G[\Delta P,P] \quad (c)$$

On this way CDCA possibilities are very improved due to the really expecting that CDCM mathematical tool for mono-cell battery may be applied to battery pack.

4. CDCM IRREVERSIBLE OVERALL ENERGY VOLUME

Electrochemical Model, Time interval and time subintervals Either Standard or CDCM task is to fill irreversible discharge data inside the defined OEV to gene-rate energy on load and energy losses parts which sum is equal to OEV reversible energy.

Standard procedure offers 'voltage vs. time' approach, i.e. time subintervals are constant that is obtained dividing discharge time interval is divided on the N steps. It means that time is independent variable and that power steps are changed. C/B response is voltage set, which is going from maximum - across minimum - to maximum, tracing 'S' shape of discharge curve. A discharge curve with time independent variable is valid for one defined discharge, only. There is no correspondence between the next discharges whatever parameter was changed.

CDCM procedure offers 'time vs. voltage' approach, i.e. time subintervals are changed while N^2 power paralle-lograms on PP remain constant. Discharge time interval is the sum of N time subintervals:

$$\Delta tinitial<\Delta tmaximum>\Delta tfinal.$$

CDCM concept in battery testing, including 3DMa-thematical Model uses the same input data as Standard concept uses.

The N^2 parallelograms '|ΔV||ΔI|' = '**ΔηΔv**' will be distributed between the four segments of PP: on load - current losses - voltage losses - CDCM losses. During

C/B discharge, step by step time intervals, parallelepipeds' altitude is positioned over PP in accordance to electrochemical model, FIGURE 3. Every time subintervals (Δt) defines altitude of CDCM energy layer in OEV. OEV and the layers have the same PP as the base, but step by step, the segments of PP are changed. Segment on load is decreased and the three other are increased. During the t_1 subinterval the power on load is reduced from N^2VI to $(N-1)^2VI$ parallelograms of the t_2 subinterval. The next layers are reduced in accordance to C/B electrochemical parameters geometrically forming energy on load 'hill' visible in FIGURE 3. The possible battery electrochemical model, applied discharge mode and regime, in a real discharge, divide the OEV into the different parts. CDCM mathematical tool is enabling to expressed the OEV' part with algebraic equations including the time and time dependent parameters. It means that C/B discharge processes may quantified, classified into reversible and irreversible, real and virtual and generated and really performed.

CDCM is enable to place time set data in 3D (time-voltage-current) space as well as 2D (time-voltage, time-current) surfaces inside OEV parallelepiped. These possibilities are visible in FIGURE 3.

One can look at time altitudes placed at Power Base (PP), by corresponded coordinates defined points as the sets of the telephone poles and imagine telethon line over their tops as the discharge curves.

A battery average cell of multi-cell battery, in serial and/or parallel connection, represents battery and is accepted as a mono-cell battery. Mono-cell PP is parallelogram, UoIo/VA and is the base of OEV, i.e. parallelepiped: UoIot/VAs. Voltage side is the same for the all load intensities, while current side depends on load, i.e. Io=Uo/Rl. The PP's four parts are: energy on load and energy losses: internal – external – CDCM polarization, Fig. 1. The 3D OEV discharge parameters are obtained du-ring real discharge in accordance to CDCM procedure.

Reversible OEV space, defined with PP (initial voltage and current) and final time is empty. When final time should be generated as the two sets: time interval and time subintervals, CDCM procedure continues to fill empty OEV space with irreversible discharge parameters. OEV empty space fills up all the data that define the discharge of C/B in all possible modes and intensities as well as those real and virtual. The next remarks explain the possible OEV contains:

- there is no any voltage/ current/ power parallelograms on PP change.
- if CDCM changes real discharge mode into different virtual mode, OEV figure is changed.
- the energy on load in FIGURE 3 is named 'hill'. Recalculted the OEV into constant current or voltage, 'hill' disapears and the new energy space is formed.

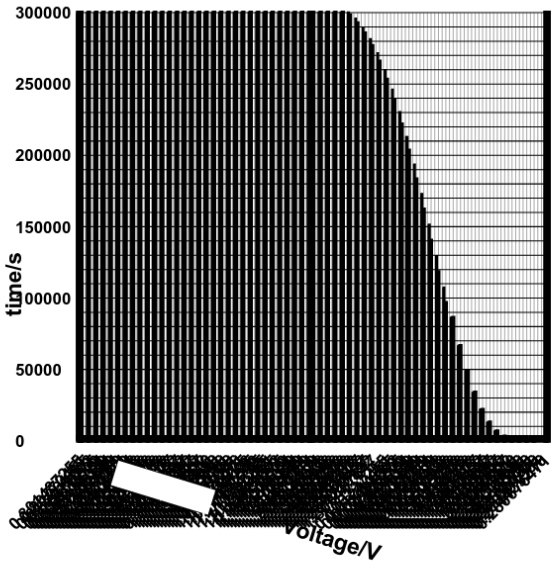

Figure 3: *OEV geometric forms/figures and parameters*

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5. CDCM EQUATIONS

C/B OEV's parallelepiped, defined in the both one calorimeter (battery and resistor) and two calorimeter (separatly battery and resistore) discharges as well as the UoIoTfinal parallelepiped is empty in the case of thermodynamical reversible discharge. There are no discharge time curves which is shown in Figure 3 as the sets of the final time altitudes, inserted abroad PP, giving 3D figure of thermodynamic reversible energy on load and energy losses. In battery testing the defining and analysing losses is the main substantial task.

The discharge time curve: time vs. voltage are defined as time interval (altitude from the begining to the marked discharge voltage. Discharge time interval is the sum of time subintervals, and there interrelations are compresed in 3P(oints) operator equation:

$$\mathbf{F}[\Delta \mathbf{t}, \mathbf{t}] = \frac{\Delta \mathbf{t}_i \mathbf{t}_{i-2}}{\Delta \mathbf{t}_{i-1} t_{i-1}} = \frac{\Delta \mathbf{P}_{\mathrm{int},i} \mathbf{P}_{\mathrm{int},i-2}}{\Delta \mathbf{P}_{\mathrm{int},i-1} \mathbf{P}_{\mathrm{int},i-1}} = \mathbf{G}[\Delta \mathbf{P}, \mathbf{P}]$$

by which CDCM has been started.

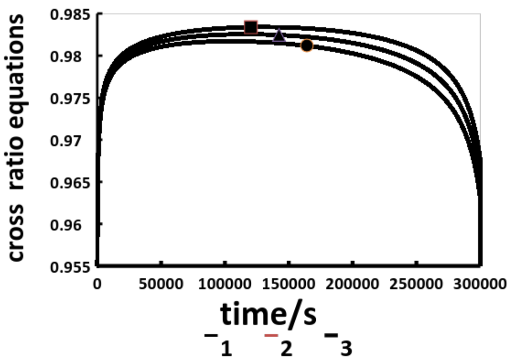

Figure 4: *Cross- ratios vs. time interval*

1: a- time, b - voltage capacity of elementary parallelepiped, c - energy of elementary

parallelepiped,

2: a-voltage capacity of energy on load, b-current capacity of energy on load,

3 - energy on load

TABLE: *Numerical parameters of cross ratios*

	1a	1b	1c	2a	2b	3
maximum	0.983398915	0.983398915	0.983398915	0.982551179	0.982551179	0.981713608
sum	301.4072815	301.4072815	301.4072815	301.0288467	301.0288467	300.6396686
average	0.953820511	0.953820511	0.953820511	0.952622933	0.952622933	0.951391356
final	0.011569496	0.011569496	0.011569496	0.011533462	0.011533462	0.011497538

In Figure 4 and TABLE *Numerical parameters of po- ssible cross ratios are cross-ratio curves of OEV's parameters. CDCM* energies of elementary parallelepiped and energy on load are shown. There are differences between time and capacities as well energy cross-ratio curves on energy on load. The question is: what is the reason for these differences? CDCM is aschieved the interrelation between the succesive an corresponded cross-ratios. Fpr energy on load it is

$$\frac{\dfrac{\Delta E_{VI,i} E_{VI,i-2}}{\Delta E_{VI,i-1} E_{VI,i-1}}}{\dfrac{\Delta Q_{I,i} Q_{I,i-2}}{\Delta Q_{I,i-1} Q_{I,i-1}}} = \frac{\dfrac{\Delta S_{V,i} S_{V,i-2}}{\Delta S_{V,i-1} S_{V,i-1}}}{\dfrac{\Delta t_i t_{i-2}}{\Delta t_{i-1} t_{i-1}}}$$

successive 3P-on load

$$\frac{\dfrac{\Delta E_{VI,i} E_{VI,i-2}}{\Delta Q_{I,i} Q_{I,i-2}}}{\dfrac{\Delta E_{VI,i-1} E_{VI,i-1}}{\Delta Q_{I,i-1} Q_{I,i-1}}} = \frac{\dfrac{\Delta S_{V,i} S_{V,i-2}}{\Delta t_i t_{i-2}}}{\dfrac{\Delta S_{V,i-1} S_{V,i-1}}{\Delta t_{i-1} t_{i-1}}}$$

corresponding 3P-on load

This equation is generalizad and is:

$$\frac{energy}{current\ capacity} = \frac{voltage\ capacity}{time}$$

which is valid for any extensive parameters inside OEV generated with self-driving discharge. The device-driving discharge need to be analysing.

6. SUMMARY CDCM TASK

CDCM analyzes C/B using general relations:

energy current capacity= voltage capacitytime

which comprises the 3P-operators energy (3D-space), capacity (2D-surface) and interval (1D-space). At one OEV elementary parallelogram the CDCM cross ratio TIMELINE

Phase 1.

In Phase 1 twelve (12) mono-cell batteries of the same type and quality should be discharged. CDCM input data are either numerically or graphical (voltage vs. time) recorded discharge data. Battery Tester Standard re-cords are suitable form. The required operating modes are constant: Resistor, Current, Voltage and Power. The overall proposed work is divided between CDCM rese-archer and Laboratory. Researcher's task is C/B analy-sing in accordance to CDCM procedure. The C/B dis-chargings is Laboratory task. Until now there is no pub-lished C/B discharge data under known discharge mo-des which discharge parameters are in time, capacities and energy correspondence on that way that these data may be interrelated and be used in multi-cell battery discharge analysing.

Laboratory tasks:

a) SELF DRIVING DISCHARGING: CONTINUAL - CONSTANT RESISTOR

The three mono-cell batteries, of same type, should be individually discharged across the resistor (three intensities of standard testing values), continual at constant temperature:

Mono - cell number:	1	2	3
Load: Resistor/Ohm:	Rl-1 <	Rl - 2 <	Rl - 3
Response: V vs. time	V - 1	V - 2	V - 3
Calculated current = V/Rl	I - 1	I - 2	I - 3

Discharge data will be sending to the researcher.

CDCM researche's task:

In accordance to CDCM procedure, the three (3) discharge curve 'voltage vs. time', of the three inten-sities (Rl-1, Rl - 2, Rl - 3), registered with Standard procedure should be recalculated into CDCM 'time vs. voltage' and analysed by CDCM procedure. CDCM recalculated these three real accordance to '*TABLE: Real and virtual discharge.* These nine CDCM generated virtual discharges should be send to 'Laboratory' that be device-driving discharged to obtain *performed virtual discharges*.

At the same time, researcher's task is to investigate the interrelations between the three self-driving vessel cells . The term 'vessel' denotes mono cell as hardwer cell, i.e. real cell. CDCM introduces the four virtual cells inside real C/B vessel. i.e. electrochemical model's: on load - voltage losses - current - losses - CDCM losses. In addition to Phase 1, i.e. three C/B discharges, the next nine device-driving is Laboratory task.

discharges into the next nine (4, 5, 6, 7, 8, 9, 10, 11 and 12) CDCM device-driving *virtual discharges* should be performed.

Phase 2.

Laboratory tasks:

a) Device driving discharge: CONTINUAL - CONSTANT CURRENT

Mono - cell number:	4	5	6
Load: Current/A	Defined by CDCM resistors discharge		
Response: V vs. time	V - 4	V - 5	V - 6

b) Device driving discharge: Rl-1 CONTINUAL - CONSTANT VOLTAGE

Mono - cell number:	7	8	9
Load: Voltage/V	Defined by CDCM resistors discharges		
Response: I vs. time	I - 7	I - 8	I - 9

c) Device driving discharge: CONTINUAL - CONSTANT POWER

Mono - cell number:	10	11	12
Load: Power/(VA)	Defined by CDCM resistors discharges		
Response:	time interval and time subintervals		

Discharge data will be send to the author.

FINAL

The three (3) mono-cell batteries were self-driving discharged across every of the three constant resistors. After that, the nine (9) mono-cell battery of the same type were device-driving discharge (constant current-voltage-power) in accordance to averaged parameters of self-driving discharge.

It is very precious experimental fund for which analysing CDCM is enable by very power full mathematical tool. A such experimental sequence as well as algebraic quantification of intensive and extensive parameters of the discharge processes in accordance to virtual and thermodynamic reversible and self -driving versus device-driving differences. In addition, this sequence is the big step to single cell discharge regime inside battery pack. And finally, it may be the first investigation to demonstrate the difference between 'voltage vs. time' and 'time va.voltage'.

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