International Journal of Innovative Studies in Sciences and Engineering Technology (IJISSET)

ISSN 2455-4863 (Online) www.ijisset.org Volume: 3 Issue: 10 | October 2017

# Multifractal Detrended Moving Average Analysis for Difference of Statistical Characteristics of AQI among Chinese Cities

**Daiyuan Li**

*College of Mathematics and Computer Science, Hunan Normal University,Changsha, 410081,P.R.China*

**Abstract:** *As the economy soars, air pollution sweeps across the China in recent years. Industrial waste gas and life stove and heating are two main source of the air pollution. Nowadays, as a indicator, air quality index (AQI) instead of the air pollution index (API), combines all kinds of pollution factors can well describe the air quality comprehensively and exactly. However, how to uncover the feature of the AQI in different areas and express the difference between them is a key point we should consider. In this paper, we employ prevalent multifractal detrended moving average analysis (MF-DMA) to investigate above the problems. Three cities, namely, average daily AQI data in recent three years of Beijing, Changsha and Zhuhai, which represent North China, Central China and South China, are chosen for our consideration. We conclude that 1) all of them possess multifractal natures by observing generalized Hurst exponent and mass index. 2) The AQI is more confused and of singularity by observing the Hölder exponent and the singularity spectrum. 3) The fluctuation of AQI is most violent between Beijing and Zhuhai that other two compared cities by using the proposed $\Delta F_q(s)$. An interesting finding is the maximum of fluctuation difference among the three cities is obtained at about 17 days, which demonstrates that fluctuation period of AQI between the North, Central and South China is about 17 days. These conclusions can provide a new insight to unveil the variation tendency of air pollution in each cities and fluctuation difference of AQI between every two cities.*

**Keywords:** *Air quality index, multifractal detrended moving average analysis, fluctuation function, singularity spectrum*

## 1. INTRODUCTION

With the rapid development of China's economy and the enlargement of city scale, air pollution is increasingly serious, which has become one of the major social problems since that the air quality of the environment is closely related with the ecological system and human health [1]. Lots of scholars have studied the problem from different perspectives [2-8]. Of which, Jassim and Coskuner [3] recognized the most pollution source impact the air quality of Bahrain Island by the method EPA and they concluded that the PM2.5 index is higher in industrial cities. Gocheva-Ilieva et al. [5] investigated the main pollutant concentration and the Bulgarian Blagoev Leningrad town of ozone concentration by using Factor analysis and Box - Jenkins methods. Lu and Fang [7] investigated the relationship between the PM2.5, PM10 and wind speed by three different distributions and they find the correlation between them is negative, which can be expressed by a special function. However, the most studies are focus on the air quality index (AQI) in single city and few of them compare the difference of the statistic characterizations between different cities. Therefore, the motivation of this work is explore the difference of AQI's singularity features between different cities. In consideration of the potential nonstationarity in AQI series, we should apply a proper statistic method to do this job.

Note that many existing methods can only deal with stationary measures of series and fail to handle the non-stationary measures. To deal with non-stationary measures, multifractal detended fluctuation analysis (MF-DFA) method was proposed by Kantelhardt [9]. After that, the MF-DFA and its modified versions have been proposed for resolving various problems in many fields [10-15]. Recently, a novel method, called multifractal detrended moving average analysis (MF-DMA) [16] has been proposed. It can also estimate the generalized Hurst exponent. Because the MF-DMA method can easily describe the multifractal nature of non-stationary series without any assumption, it is widely used in analysis of time series [17]. Some empirical analyses show that the MF-DMA is superior to MF-DFA in certain circumstances [18].

Although the existing literatures have confirmed that AQI series is in line with the multifractal characteristics, there are rare reports about the specific fractal feature and its genesis of the AQI in different cities. In this paper, we use the popular MF-DMA method to study the AQI in three of the typical pollution cities in China. We first investigate multifractal nature of the AQI. And then reveal differences of the dynamic characteristic of the AQI by multifractal singularity spectrum. Finally, we access the fluctuation difference of AQI among the three cities by a new proposed quantity.

## 2. METHOD AND DATA

### 2.1Multifractal detrended moving average analysis (MF-DMA)

Let's remind the MF-DMA's procedure as follows, which contains five steps.

Step 1: For a time series $x(t), t=1,\cdots,N$ ,Construct the cumulative sums

$$y(t)=\sum_{i=1}^{t}x(i), t=1,\cdots,N, \qquad (1)$$

Step 2: Calculate the moving average function $\tilde{y}(t)$ in a moving window

$$\tilde{y}(t)=\frac{1}{s}\sum_{k=-\lfloor (s-1)\theta \rfloor}^{\lceil (s-1)(1-\theta)\rceil} y(t-k), \qquad (2)$$

where, s is the window size, $\lfloor x \rfloor$ is the largest integer no greater than $x$, $\lceil x \rceil$ is the minimum integer not less than $x$. Hence, the moving average function considers $\lceil (s-1)(1-\theta)\rceil$ data points in the past and $\lfloor (s-1)\theta \rfloor$ points in the future. $\theta$ is the position parameter variables in [0, 1]. In our work, we took $\theta=1$. Hence, $\tilde{y}(t)$ is considered the trend of $s-1$ data points in the future.

Step 3: By removing the moving average function $\tilde{y}(t)$ from $y(i)$, and obtain the residual error sequence i can be obtained:

$$\varepsilon(i)=y(i)-\tilde{y}(i), \qquad (3)$$

Where $s-[(s-1)\theta]\le i\le N-[(s-1)\theta]$.

Step 4: Divide the $\varepsilon(i)$ into $N_s=[N/s-1]$ un-overlap segments with the same size $s$. Each segment can be denoted by $\varepsilon_v(i)=\varepsilon(l+i)$ for $1\le i\le s$ , where $l=(v-1)s$ The root-mean-square function $f(v,s)$ is calculated by:

$$f^2(v,s)=\frac{1}{s}\sum_{i=1}^{s}\varepsilon_v^2(i), \qquad (4)$$

The $q^{th}$ order overall fluctuation $F_q(s)$ is determined as follows:

$$F_q(s)=\left[\frac{1}{N_s}\sum_{v=1}^{N_s}f^q(v,s)\right]^{1/q}, q\ne 0, \qquad (5)$$

When $q=0$, according to the $L'H\hat{o}spital$ law, we have

$$\ln[F_0(s)]=\frac{1}{N_s}\sum_{v=1}^{N_s}\ln[f^2(v,s)], \qquad (6)$$

The $F_q(s)$ expresses different fluctuations through the different order $q$, which describes smaller fluctuations with smaller $q$'s and larger fluctuations with bigger $q$'s . In general, exponent $q<0$ magnifies the contribution to $F_q(s)$ from small amplitude fluctuations and $q>0$ magnifies the contribution from large amplitude fluctuations. To access difference of fluctuation degree between two series quantitatively, we define a new quantity as follows:

$$\Delta F_q(s)=\log F_q^{(1)}(s)-\log F_q^{(2)}(s), \qquad (7)$$

here, $F_q^{(1)}(s)$ and $F_q^{(2)}(s)$ denote $q^{th}$ order fluctuation function of series $\{x_1(t)\}$ and series $\{x_2(t)\}$ , respectively.

Step 5: Varying the size of each segment $s$, we can determine the power-law relation between the function $F_q(s)$ and the size scale s as follows

$$F_q(s)\sim s^{h(q)}, \qquad (8)$$

where, $h(q)$ can be obtained by fitting $F_q(s)$ and $s$ in log-log plot. The exponent $h(q)$ describes the volatility persistence of AQI accumulated deviation, which generally depends on $q$ . When $h(q)=0.5$ , the sequence has the scale invariance, which suggests that the original AQI is an independent random process. When $0<h(q)<0.5$, it indicates that the original AQI have no persistence but anti-persistence. The $h(q)$ within the scope of (0.5, 1) means the AQI has the long-range correlation, and who is stationary. When $h(q) > 1$, the time series is still persistent with long-range correlation, but who is non-stationary.

Mass exponents $\tau(q)$ can be used to get by the obtained $h(q)$

$$\tau(q) = qh(q) - 1. \qquad (9)$$

The nonlinear relationship between the $\tau(q)$ and $q$ suggests that the AQI possesses multifractal nature. From the $\tau(q)$ ,one can obtain another group important parameters, namely, $Lipschitz - Hölder$ exponent $\alpha(q)$ and singularity spectrum $f(\alpha)$ through the $Legendre$ transformation, as follows

$$\begin{cases} \alpha(q) = \tau'(q) = h(q) + qh'(q) \\ f(\alpha) = q\alpha(q) - \tau(q) \end{cases}, \qquad (10)$$

$\alpha(q)$ describes local singularity of the series $\{x(t)\}$ while $f(\alpha)$ denotes the global singularity. The smaller of the $\alpha(q)$, the more singularity will be expected. For a one-dimensional measure, the maximum of $f(\alpha)$ is 1. We mark $\alpha^*$ when $f(\alpha) = 1$. Another group quantity used to characterize the singularity of the studied series is the gap of the $\alpha(q)$ and width of the singular multifractal spectrum, which can be determined by

$$\Delta\alpha = \alpha_{\max} - \alpha_{\min}, \Delta f = f(\alpha_{\min}) - f(\alpha_{\max}), \quad (11)$$

$\alpha_{\max}$ and $\alpha_{\min}$ are the maximum value of the $\alpha(q)$ and the minimum value of the $\alpha(q)$, respectively (for instance, we took $q = -5$ to 5 in our work). The index $\Delta\alpha$ is considered as a measure indicator which is used to indicate absolute magnitude of AQI volatility. The bigger of $\Delta\alpha$ is, the smaller even distribution of probability measure is, and the more violent AQI fluctuations will usually be expected. $\Delta f$ spectra is Hausdorff dimension of the measure object, and the Hausdorff dimension is an indicator used to measure degree of confusion. Specifically ,if $\Delta f$ , it shows that AQI running above the mean AQI is more than the AQI running below the mean value and the AQI shows strong performance, and the multifractal spectrum presents the "left hook" shape; On the contrary, if $\Delta f < 0$, the AQI shows weak performance, and the multifractal spectrum presents the "right hook" shape. In this regard, both $\Delta\alpha$ and $\Delta f$ are also important multifractal parameters, which are employed in our work.

### 2.2 Data description

In this study, we focus our interest on the difference of AQI's series among North China, Central China and South China. To this end, we choose one city for those areas, respectively, namely, Beijing $(39.93^\circ N, 116.39^\circ E)$, Changsha $(28.23^\circ N, 112.94^\circ E)$ and Zhuhai $(22.27^\circ N, 113.58^\circ E)$. Each chosen city can represent the environmental conditions of the three studied areas. And all of them have been greatly affected by heavy smog in recent years but in varying degrees. We record daily average data of AQI series of these three cities from December 1, 2013 to November 30, 2016 from the Ministry of Environmental Protection of the People's Republic of China (http://datacenter.mep.gov.cn), which are shown in Fig. 1, and their general statistics are list in Table 1. As seen from the Fig. 1. One can easy find that the index of AQI is higher in Beijing than that in Changsha and Zhuhai. In addition, the fluctuation of AQI in Beijing is most severe among the three cities, which demonstrates that the air pollution of Beijing is most serious and the environment has changed dramatically as well. This conclusion can also be supported by the Table 1.

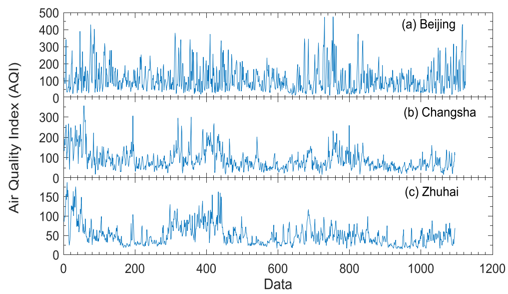


**Fig 1:** *The AQI of (a) Beijing, (b) Changsha and (c) Zhuhai from Dec. 1, 2013 to Nov. 30, 2016.*

**Table 1:** *General statistics of the AQI series of Beijing, Changsha and Zhuhai*

| | Mean | Standard deviation | Range | Kurtosis | Skewness | Coefficient of variation |
|---|---|---|---|---|---|---|
| Beijing | 111.7353 | 76.1126 | 460 | 5.4043 | 1.4839 | 0.6812 |
| Changsha | 88.9406 | 49.9035 | 338 | 6.4599 | 1.6654 | 0.5611 |
| Zhuhai | 51.4201 | 27.7330 | 172 | 5.8869 | 1.5306 | 0.5393 |

## 3. DATA ANALYSIS AND DISCUSSION

At first, we would like to investigate multifractal nature of the AQI series in the three studied cities. To do this, by using (8) and (9), the generalize Hurst exponent $h(q)$ and the mass index $\tau(q)$ are plotted in Fig. 2. If $\tau(q)$ is nonlinear in $q$, that is, $h(q)$ is not independent of $q$, then the series possesses the multifractal nature. As expected, we find that both of them possess the multifractal nature due to the fact that that $\tau(q)$ is nonlinear in $q$ (inset plot in each sub-figure), indicated by the fact that $h(q)$ depends on $q$.

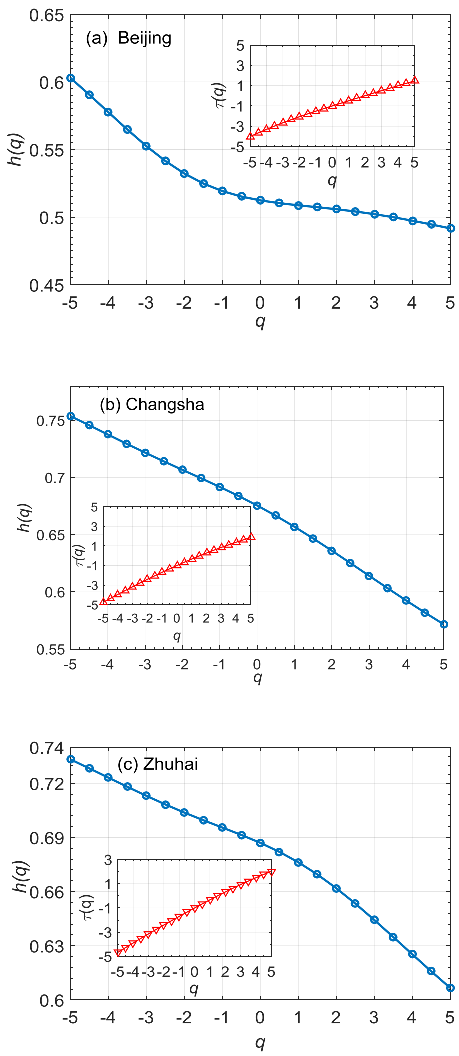


**Fig 2:** *Multifractal nature in the power-law of dependence of τ(q) and h(q) on q. (a) is for Beijing, (b) is for Changsha and (c) is for Zhuhai.*

Now, we employ the $Lipschitz-Hölder$ exponent $\alpha(q)$ and singularity spectrum $f(\alpha)$ to detect the difference of the AQI's multifractal nature among the three studied cities. By using the Eqs. (10) and (11), the curves of $\alpha - f(\alpha)$ of Beijing, Changsha and Zhuhai are shown in Fig. 3, and the singularity of the multifractal features in the three cities are reported in Table 2.

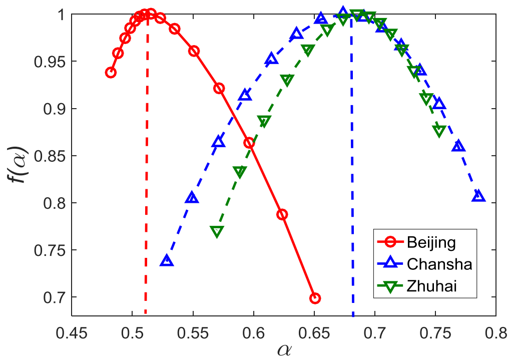


**Fig 3:** *Curves of α-f(α) of Beijing, Changsha and Zhuhai.*

**Table 2:** *The singularity of the multifractal features in Beijing, Changsha and Zhuhai*

| | $\alpha_{max}$ | $\alpha_{min}$ | α* | Δα | Δf(α) |
|---|---|---|---|---|---|
| Beijing | 0.6509 | 0.4823 | 0.5152 | 0.1686 | 0.2399 |
| Changsha | 0.7852 | 0.5283 | 0.6737 | 0.2570 | -0.0680 |
| Zhuhai | 0.7530 | 0.5695 | 0.6851 | 0.1835 | -0.1065 |

Seen from Fig. 3, the $f(\alpha)$ spectra of AQI Beijing is shown as "left hook" but "right hook" in Changsha and Zhuhai. The interesting finding is explained that tendencies of the AQI values are strengthened in Beijing but weakened in Changsha and Zhuhai, which is consistent with the fact of that AQI of Beijing running above the mean value is more than the AQI running below it, the situation in Changsha and Zhuhai is the reverse, as shown in Fig. 1. The $\Delta f(\alpha)$ in Beijing is larger than other two cities also indicates that the AQI is of much confusion in Beijing. Another evidence supports this is the proposed $\alpha^*$, which is obtained when the $f(\alpha^*)=1$. The $\alpha^*$ together with $\alpha_{\max}$ and $\alpha_{\min}$ of Beijing is smaller than that of Changsha and Zhuhai significantly, which shows that the singular AQI values are more likely to appear in Beijing at some local time scales. What's more, we find the gap of local singularity ($\Delta\alpha$) in Changsha is larger than that in Zhuhai although their local singularities ($\alpha_{\max}$, $\alpha_{\min}$ and $\alpha^*$) are similar, which illustrates that the transformation of air quality in Central China at local time window is more remarkable that in South China.

As the last point we concern, we would like to access the fluctuation of AQI at different time scales in the three studied cities quantitatively. In this connection, we used the proposed $\Delta\log(F_q(s))$ (defined in Eq.(8)). Here, we take $q=-2$ and q = 2 to investigate the smaller amplitude fluctuations and the larger amplitude fluctuations, respectively. The results are shown in Fig.4. Seen from the Fig.4, the trends of the $\Delta\log(F_q(s))$ respective with $s$ are similar for $q=-2$ and $2$. It is explained that the fluctuation difference among the three cities is similar in smaller amplitude and larger amplitude fluctuations. As expected, the difference between Beijing and Zhuhai is more significantly than other two compared cities, which indicates that the AQI is of higher volatility compared to the other two cities. An interesting finding is that the maximum of fluctuation difference among the three cities is obtained at about 17 days, which demonstrates that fluctuation period of AQI between the North, Central and South China is about 17 days.

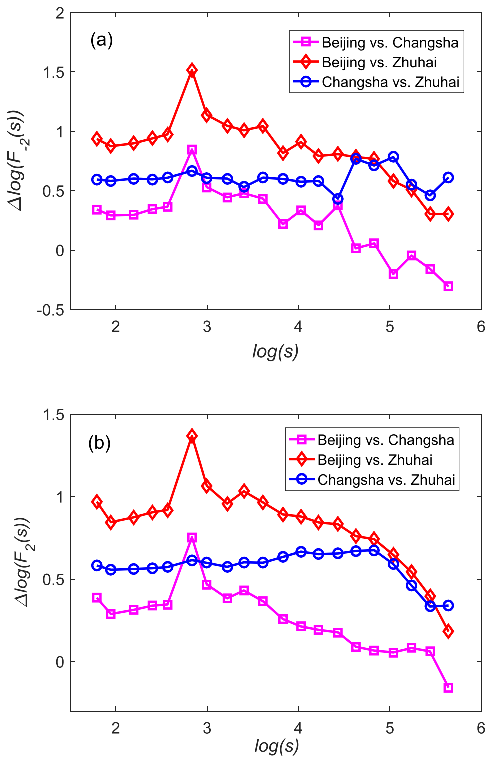


**Fig 4:** *Difference of fluctuation functions between Beijing and Changsha, Beijing and Zhuhai, and Changsha and Zhuhai, respectively. (a) is at smaller amplitude fluctuations and (b) is at larger amplitude fluctuations.*

## 4. CONCLUSIONS

In this work, by employing the prevalent physical statistic method—multifractal detrended moving average analysis (MF-DMA), we investigate the average daily AQI of three typical Chinese polluted cities, namely, Beijing, Changsha and Zhuhai in recent three years. At first, we check the three AQI series possess multifractal natures by observing the dependence of $h(q)$ and $\tau(q)$ on $q$. And then, by calculating the multifractal feature parameters and drawing the curves of $\alpha - f(\alpha)$, we find tendencies of the AQI values are strengthened in Beijing but weakened in Changsha and Zhuhai. The AQI is more confused in Beijing than other two studied cities. Finally, by calculating the proposed , $\Delta\log(F_q(s))$ we find Beijing is of most violent fluctuation. And the fluctuation difference between Beijing and Zhuhai is most significant. An interesting finding is that the maximum of fluctuation difference among the three cities is obtained at about 17 days.

## REFERENCES

[1] Chak K. Chan, X. Yao, Air pollution in mega cities in china, Atmospheric Environment ,vol. 42, pp. 1–42, 2008.

[2] L. J. Han, W. Q. Zhou, and W. F. Li, Increasing impact of urban fine particles (pm2.5) on areas surrounding chinese cities. Scientific Reports 5, 12467 (2015).

[3] M. S. Jassim and G. Coskuner, Assessment of spatial variations of particulate matter (PM10 and PM2.5) in Bahrain identified by air quality index (AQI), Arabian Journal of Geosciences, vol.10,pp.19-25, 2017.

[4] E. Jarauta-Bragulat, C. Hervada-Sala and J. J. Egozcue, Air Quality Index Revisited from a Compositional Point of View, Mathematical Geosciences, vol. 48, pp. 1-13, 2016.

[5] S. G. Gocheva-Ilieva, A. V. Ivanov, D. S. Voynikova, et al. Time series analysis and forecasting for air pollution in small urban area: an SARIMA and factor analysis approach, Stochastic Environmental Research and Risk Assessment, vol. 28, pp. 1045-1060, 2014.

[6] L. J. Han, W. Q. Zhou and W. F. Li, City as a major source area of fine particulate (pm 2.5) in china.,Environmental Pollution, vol.206, pp.183–187 ,2015.

[7] H. C. Lu and G. C. Fang, Estimating the frequency distributions of PM10 and PM2.5 by the statisticsof wind speed at Sha-Lu , Taiwan, Science of the Total Environment, vol. 298, pp.119-130, 2002.

[8] C. H. Shen, A new detrended semipartial cross-correlation analysis: Assessing the important meteorological factorsaffecting API, Physics Letters A,vol. 379, pp. 2962–2969, 2015.

[9] J. W. Kantelhardt, S. A. Zschiegner, E. K. Bunde, et al, Multifractal detrended fluctuation analysis of non-stationary time series, Physica A ,vol.316, pp.87-114, 2002.

[10] F. Wang, G.-P. Liao, J.-H. Li, Multifractal detrended fluctuation analysis for clustering structures of electricity price periods, Physica A ,vol. 392, pp. 5723-5734,2013

[11] M. S. Movahed, G.R. Jafari, F. Ghasemi, et al., Multifractal detrended fluctuation analysis of sunspot time series, Journal of Statistical Mechanics:Theory and Experiment, vol. 2, pp.1-15,2006.

[12] Y. Zhou and Y. Leung, Multifractal temporally weighted detrended fluctuation analysis and its application in the analysis of scaling behavior in temperature series, Journal of Statistical Mechanics: Theory and Experiment,vol.6, pp. 06021, 2010

[13] B. Podobnik, D. Horvatic, A.M. Petersen, H.E. Stanley, Cross-correlations between volume change and price change, Social Science Electronic Publishing, vol. 106 ,pp.22079-22084, 2009

[14] L. Hedayatifar, M. Vahabi, G.R. Jafari, Coupling detrended fluctuation analysis for analyzing coupled non- stationary signals, Physical Review E,vol. 84: 01138, 2011.

[15] K. Hu, Z. Chen, P.Ch. Ivanov, P. Carpena, H.E. Stanley, Effect of trends on detrended fluctuation analysis, Physical Review E ,vol.64 :011114,2001.

[16] A. Y. Schumann and J. W. Kantelhardt, Multifractal moving average analysis and test of multifractal model with tuned correlations, Physica A ,vol. 390, pp.2637-2654,2011.

[17] Y. Wang, C. Wu, and Z. Pan, Multifractal detrending moving average analysis on the US Dollar exchange rates, Physica A,vol.390,pp.3512-3523, 2011.

[18] Y. H. Shao, G. F. Gu, Z. Q. Jiang, et al., Comparing the performance of FA, DFA and DMA using different synthetic long-range correlated time series, Scientific reports, vol.2 ,pp.835, 2012.

## AUTHOR'S BIOGRAPHY

**Daiyuan Li:**he is a postgraduate student of Hunan Normal University