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COMPABILITY AND CONTRACTION MAPPINGS IN HILBERT SPACE

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Abstract: *In the present paper, I prove the existence of common fixed point and contraction mapping in Hilbert spaces by iretates.*

Keywords and Phrases: *Hilbert space, common fixed point, Contraction, Cauchy sequence.*

AMS Subject classification: *Primary 47H10, Secondary 54H25*

1. INTRODUCTION

In recent years some fixed points of various type of compability mapping in Hilbert space were obtained by Browder[1],Browder and Petryshyn[2], Hicks and Huffman [3], Jungck[4].

2. PRELIMINARIES

2.1 **NORM** : A norm on X is a real-valued function ||.|| : X→R defined on X such that for any x, y ∈ X and for all λ∈K

(a) ||x|| = 0 if and only if x = 0

(b) ||x+y|| ≤ ||x|| + ||y||

(c) ||λx|| = |λ| ||x||

2.2 **NORMED LINEAR SPACE** : It is a pair (X, ||.||) consisting of a linear space X and a norm ||.||. We shall abbreviate normed linear space as nls.

2.3 CAUCHY SEQUENCE : A Sequence $\{x_n\}$ in a nls X is a Cauchy sequence if for any given ε > 0, there exist n_0 ∈ N such that $||x_m - x_n|| < \varepsilon$ for m, n ≥ n_0

2.4 CONVERGENCE CONDITION IN NLS : A sequence $\{x_n\}$ in a nls X is said to be Convergent to x ∈ X if for any given ε > 0, ∃ n_0 ∈ N such that $||x_n - x|| < \varepsilon$ for n ≥ n_0

2.5 COMPLETENESS : A nls X is said to be complete if for every Cauchy Sequence in X converges to an element of X.

2.6 BANACH SPACE : A Banach Space (X, ||.||) is a complete nls.

2.7 INNER PRODUCT SPACE: Let X be a linear space over the scalar field C of complex numbers. An inner product on X is a function (. , .) : XxX → C which satisfies the following conditions

(a) (x, y) = $\overline{(y, x)}$ for x, y ∈ X

(b) (λx + μy, z) = λ (x, z) + μ (y, z) for λ, μ ∈ C, x, y, z ∈ X

(c) (x, x) ≥ 0 ; (x x) = 0 iff x = 0

2.8 LAW OF PARALLELOGRAM: If x and y are any two elements of an inner product space X then $||x + y||^2 + ||x-y||^2 = 2||x||^2 + 2||y||^2$

X is nearly equal y then $||x + y||^2 = 2||x||^2 + 2||y||^2$

2.9 HILBERT SPACE: An infinite dimensional inner product space which is complete for the norm induced by the inner product is called Hilbert Space.

3. MATERIAL AND METHOD

Theorem: Let C be a closed subset of a Hilbert space H.A,B.S,T:C→C are contractive and continuous map of C , then A,B,S,T converges to p. If A,B.S,T satisfying the following condition

$$\|Tx - Sy\|^2 \le a\|Sy - Ax\|^2 + b\|x - Tx\|^2 + c\|x - Ty\|^2 + d\|y - By\|^2 + e\|Sx - By\|^2$$

Then A,B,S,T have a common fixed point p if a+b+3c+d+e<1.

4. RESULT AND DISCUSSION

Proof of theorem:

Consider $x=x_n$ and $y=x_{n+1}$ then

$$\|Tx_n - Sx_{n+1}\|^2 \le a\|Sx_{n+1} - Ax_n\|^2 + b\|x_n - Tx_n\|^2 + c\|x_n - Tx_{n+1}\|^2 + d\|x_{n+1} - Bx_{n+1}\|^2 + e\|Sx_n - Bx_{n+1}\|^2$$

Also suppose that

$Tx_n=Ax_n=Bx_n=Sx_n=x_{n+1}$ and

$Q_{n+1} = \|x_{n+1} - x_{n+2}\|^2$

Then above reduces to

$\|x_{n+1} - x_{n+2}\|^2 \le a\|x_{n+2} - x_{n+1}\|^2 + b\|x_n - x_{n+1}\|^2$

$+c\|x_n - x_{n+2}\|^2 + d\|x_{n+1} - x_{n+2}\|^2 + e\|x_{n+1} - x_{n+2}\|^2$

$\Rightarrow \|x_{n+1} - x_{n+2}\|^2 \le a\|x_{n+2} - x_{n+1}\|^2 + b\|x_n - x_{n+1}\|^2$
$+2c\|x_n - x_{n+1}\|^2$

$+2c\|x_{n+1} - x_{n+2}\|^2 + d\|x_{n+1} - x_{n+2}\|^2 + e\|x_{n+1} - x_{n+2}\|^2$

$\Rightarrow Q_{n+1} = a Q_{n+1} + b Q_n + 2c Q_n + 2c Q_{n+1}$

$+d Q_{n+1} + e Q_{n+1}$

$\Rightarrow (1-a-2c-d-e) Q_{n+1} = (b+2c) Q_n$

$$\Rightarrow Q_{n+1} = \frac{b+2c}{1-a-2c-d-e} Q_n$$

$$\Rightarrow Q_{n+1} \le k Q_n \quad \text{where } k = \frac{b+2c}{1-a-2c-d-e}$$

$\Rightarrow Q_{n+1} \le k^2 Q_{n-1}$

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$\Rightarrow Q_{n+1} \le k^n Q_0$

Here $Q_0 = \|x_1 - x_0\|^2$ is a Cauchy sequence and also convergent ,then it is easy to show that at the point p .Which is common fixed point of A,B,S,T,hence the theorem.

Also I claim that above is a compability of type(A)

$\|TSx - SSy\|^2 \le a\|SSy - TAx\|^2 + b\|Tx - TSx\|^2$

$+c\|Tx - TSy\|^2 + d\|Ty - SBy\|^2 + e\|SSx - TBy\|^2$

When $a+b+3c+d+e<1$

Let $Tx_n=Ax_n=Bx_n=Sx_n=x_{n+1}$ and

$Q_n = \|x_{n+1} - x_{n+2}\|^2$ then

$\Rightarrow Q_{n+1} = a Q_{n+1} + b Q_n + 2c Q_n + 2c Q_{n+1}$

$+d Q_{n+1} + e Q_{n+1}$

$$\Rightarrow Q_{n+1} = \frac{b+2c}{1-a-2c-d-e} Q_n$$

$$\Rightarrow Q_{n+1} < k Q_n \text{ where } k = \frac{b+2c}{1-a-2c-d-e}$$

$\Rightarrow Q_{n+1} \le k^n Q_0$

Here $Q_0 = \|x_1 - x_0\|^2$ is a Cauchy sequence and also convergent at the point p .It is easy to show that $Tx_n=Ax_n=Bx_n=Sx_n=x_{n+1}$ = p at limit $n\to\infty$. Therefore A,B,S,T has a common fixed point p and also I have proved that it is a compability of type(A) ,hence the theorem is completed.

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