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Kannan Type Contraction Mapping in Fuzzy 2-Metric Space

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Abstract: *The aim of present paper is to obtain a fixed point by contraction in fuzzy 2-metric space Kannan type. My purpose is generalize the several known results.*

Keywords and Phrases: *fuzzy 2-metric spaces, contraction mapping, fixed point.*

AMS Subject Classification (2000): *54H25, 47H10.*

1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [12] in 1965, who introduced the concept of fuzzy set. George and Veeramani [3] modified the notion of fuzzy metric spaces by introducing the concept of continuous t – norm. Now define fuzzy 2-metric space. The fuzzy metric space was introduced by Kramosil and Michalek [5], Sao, G.S.and Shrivastava, U.K. Introduce the notion of compability of type (D)[10]

2. PRELIMINARIES

DEFINITION 2.1([12]) Let X be any set. A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition(2.2)[3]:Abinary operation : $[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm if $\{[0,1],*\}$ is an abelian topological monoid with unit 1 such that $a*b\le c*d$ whenever $a\le c$ and $b\le d$, $a,b,c,d\in[0,1]$

Definition(2.3)[3]:A **fuzzy metric space** is a space X in which for each triple of points x,y,z there exits a real function M(x,y,t) such that :

(i)to each pair of distinct points x,y in there exists a point t in[0,1] then $M(x,y,t)\neq 0$;

(ii) $M(x,y,t) = 1$, when x,y are equal;

(iii) $M(x,y,t)= M(y,x,t)$;

(iv) $M(x,y,t_1+t_2+t_3)$

$\geq M(x,y,t_1) * M(x,y,t_2) * M(x,y,t_3)$,

for all x,yin X and t_1,t_2,t_3 in [0,1].It is easily seen that M is non -negative.

Definition(2.4)[3]:A sequence$\{x_n\}$in a fuzzy metric space(X,M,∗)is said to be **convergent** with lim x in X if

$\lim_{n\to\infty} M(x_n,x,t)=1$,for all z inX and t>0.

Definition(2.5)[3]:A sequence $\{x_n\}$ in a fuzzy metric space (X,M,∗) is said to be a **Cauchy sequence** if

$\lim_{m,n\to\infty} M(x_m,x_n,t)=1$,for all z inX and t>0.

Definition(2.5)[3]:A fuzzy metric space (X,M,∗)is said to be **complete** fuzzy **metric space** if every Cauchy sequence in X is convergent.

DEFINITION 2.6([1]) Two self-mappings A and S of a fuzzy metric space (X,M,∗) are said to be compatible if,

$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence such that,

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.7([4]) Two self-mappings A and S of a fuzzy metric space (X,M,∗) are said to be weakly compatible if, they commute at coincidence points. That is, $Ax=Sx$ implies that $ASx=SAx$ for all x in X.

DEFINITION 2.8([2]) Two self-mappings A and S of a metric space (X,M,∗) are said to be subcompatible if, $\lim_{n\to\infty} M(ASx_n, SAx_n,t) = 1$ whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.9([6]) Two self-mappings A and S of a fuzzy metric space (X,M,∗) are said to be reciprocally continuous if,

$\lim_{n\to\infty} M(ASx_n, Ax, t) = 1$ and $\lim_{n\to\infty} M(SAx_n, Sx, t) = 1$

whenever $\{x_n\}$ is a sequence such that,

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

It is noted that if A and S are both continuous, they are obviously reciprocally continuous but the converse need not be true. For this, we have the following example:

EXAMPLE 1.1([6]) Consider two mappings A and S defined on X = [2, 20] with the usual Euclidean metric d, defined by the rule A2 = 2, A*x* = 3 if *x* > 2 and S2 = 2, S*x* = 6 if *x* > 2. Then, A and S are reciprocally continuous but are not continuous.

3. MATERIAL AND METHOD

Theorem(3.1):Let (X,M,∗) be a fuzzy 2-metric space and let T be continuous self mappings of X , then $\{Tx_n\}$ converges to p. If T satisfying the following condition

$M(Tx,Ty,z,t) \le aM(x,Tx,z,t)$

$+bM(y.Ty,z,t)+cM(x,Ty,z,t)$

$+dM(y,T^2x,z,t)+eM(x,y,z,t)$

s.t. $x,y,z,t \in [0,1]$ and $a+b+2c+d+e<1$

Then T has a fixed point p.

4. RESULT AND DISCUSSION

Proof of theorem:Consider $Q_n=M(x_n,x_{n+1},z,t)$ suppose $x=x_n$, $y=x_{n+1}$ then we have

$Q_{n+1} \le aQ_n+bQ_{n+1}+c(Q_n+Q_{n+1})$

$+dQ_{n+1}+eQ_n$

$(1-b-c-d)\ Q_{n+1} \le (a+c+e)\ Q_n$

$$Q_{n+1} \le \frac{a+c+e}{1-b-c-d}\ Q_n$$

$Q_{n+1} \le k\ Q_n$

when $k=\dfrac{a+c+e}{1-b-c-d}<1$,

hence $a+b+2c+d+e<1$

$\Rightarrow\ Q_n \le k\ Q_{n-1}$

$\Rightarrow\ Q_{n+1} \le k^2\ Q_{n-1}$

$\Rightarrow\ Q_n \le k^n\ Q_0$

But $Q_0=M(x_0,x_1,z,t)$ which is a Cauchy fuzzy metric space .So Qn be Cauchy fuzzy metric space .Also we can easily prove it will be convergent.

$M(Tx_n,p,z,t) \le M(Tx_n,p,z,t/2)* \ M(Tx_n,p,z,t/4)* M(Tx_n,p,z,t/8)*\ldots\ldots$ $M(Tx_n,p,z,t/2^n)$

$\Rightarrow M(Tx_n,p,z,t) \to 1$

Therefore it should be converge at the point p .Also we can show point p is unique.

5. CONCLUSION

As Q_n or $\{Tx_n\}_{n=1}^{\infty}$ is a Cauchy sequence, then $Tx_n=p$.

So $M(Tx_n,p,z,t) \to 1$

Therefore T has a fixed point p.

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