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Common Fixed Point Theorem by Weakly Compatible Mappings

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Abstract: *In this paper we prove common fixed point theorem by weakly compatible mappings in fuzzy b-metric spac .*

1. INTRODUCTION

Fixed point theory has got wide applications in different branches of mathematics.In 1922, the work of S. Banach known as the Banach Contraction Principle, laid the foundation for studies of fixed point theory. Since then,many mathematicians have extended and generalised the work of Banach in different directions. In [8], Bakhtin introduced the concept of a b-metric space as a generalisation of a metric space and proved the contraction mapping theorem in a b-metric space. Since . Huang and Zang [3], generalising the notion of a metric space by replacing the set of real numbers by ordered normed spaces, In [7], Hussein and Shah introduced cone b-metric spaces as a generalisation of b-metric spaces and cone metric spaces, established some topological properties in such spaces and improved . Cone b-metric spaces play a useful role in fixed point theory.

2. PRELIMINARIES

Let E be a real Banach space and P a subset of E. Then P is called a cone if

(i) P is closed, non-empty and satisfies $P \neq \{0\}$,

(ii) $ax + by \in P$ for all $x; y \in P$ and non-negative real numbers a; b,

(iii) $x \in P$ and $-x \in P \Rightarrow x = 0$, i.e.$P \cap (-P) = 0$.

Definition 2.1 ([3]). Let X be a non empty set. Suppose that the mapping $d : X \times X \rightarrow E$ satisfies:

(d1) $\theta \preceq d(x;y)$ for all $x;y \in X$ and $d(x; y) = \theta$ if and only if $x = y$,

(d2) $d(x; y) = d(y; x)$ for all $x; y \in X$,

(d3)$d(x;y) \preceq d(x;z) + d(z; y)$ for all $x; y; z \in X$.

Then d is called a cone metric on X and (X,d) is called a cone metric space.

Definition 2.2 ([3]). Let X be a non empty set and $s \geq 1$ be a given real number. Suppose that the mapping $d : X \times X \rightarrow E$ satisfies:

(d1) $\theta \preceq d(x;y)$ for all $x;y \in X$ and $d(x; y) = \theta$ if and only if $x = y$,

(d2) $d(x; y) = d(y; x)$ for all $x; y \in X$,

(d3) $d(x; y) \preceq s[d(x; z) + d(z; y)]$ for all $x; y; z \in X$.

Then, d is called a cone b-metric on X and (X; d) is called a cone b-metric space.

Definition 2.3 ([3]). Let (X, d) be a cone b-metric space. The sequence $\{x_n\}$ in X is said to be:

(a) a convergent sequence if for every $c \in E$ with $0 \ll c$, there is $n_0 \in N$ such that for all $n \geq n_0$,$d(x_n; x) \ll c$ for some $x \in X$. We denote this by $\lim_{n\to\infty} x_n = x$,

(b) a Cauchy sequence if for all $c \in E$ with $0 \ll c$, there is $n_0 \in N$ such that $d(x_m; x_n) \ll c$,

for all $m; n \geq n_0$,

(c) a cone b-metric space (X; d) is said to be complete if every Cauchy sequence in X is convergent in X.

Definition 2.4 ([4]). Let (X; d) be a metric space, k a positive integer,$T : X^k \rightarrow X$ and $f : X \rightarrow X$ be mappings.

(a) An element $x \in X$ is said to be a coincidence point of f and T if and only if $f(x) = T(x, x,\dots x)$. If $x = f(x) = T(x, x,\dots x)$, then we say that x is a common fixed point of f and T. If $w = f(x) =T(x, x,\dots x)$, then w is called a point of coincidence of f and T.

(b) Mappings f and T are said to be commuting if and only if $f(T(x, x, \dots x)) = T(fx, fx,\dots fx)$ for all $x \in X$.

(c) Mappings f and T are said to be weakly compatible if and only if they commute at their coincidence points.

Definition 2.5 A fuzzy metric space $(X,M,*)$ is said to be weak **contraction** fuzzy b-**metric space** if there exit $s\in(0,1)$ and $G\geq 0$

$M(Ax,Ay,t) \geq sM(x,y,t)*GM(y,Ax,t)$

OR $M(Ax,Ay,t) \geq sM(x,y,t)*GM(x,Ay,t)$

for all $x,y\in X$ and $A:X\rightarrow X$.

3. MATERIAL AND METHODS

Theorem 3.1 Let $(X,M,*)$ be a complete fuzzy b-metric space,for any positive integer k let $A,B: X^k\rightarrow X$ and $g: X \rightarrow X$ be mappings satisfying

$A(X^k)\cup B(X^k)\subset g(X)$ (3.1)

$$\max_{\substack{\alpha=A\vee B\\ \beta=A\vee B}} \{M(\alpha(x_1,x_2,x_3,\ldots x_k),\beta(x_2,x_3,\ldots\ldots x_{k+1}),t)$$

$\preceq \mu\Psi(M(gx_1,gx_2,t),M(gx_2,gx_3,t)\ldots M(gx_k,gx_{k+1},t))$

for all $x_1,x_2\ldots\ldots x_{k+1}\in X, s^k\mu\in(0,1)$ (3.2)

If $g(X)$ is complete then there exit elements $x_1,x_2,x_3,\ldots\ldots x_{k+1}$ in X s.t. $g(x_{k+1})= B(x_1,x_2,x_3,\ldots\ldots x_k)$ (3.3)

If R is the upper bound of the set $\{M(\frac{gx_1,gx_2,t}{\theta})$, $M(\frac{gx_2,gx_3,t}{\theta^2})\ldots\ldots\ldots M(\frac{gx_k,gx_{k+1},t}{\theta^k})$

Where $\theta=\mu^{\frac{1}{k}}$ (3.4)

If the pairs (g,A) and (g,B) are weakly compatible pairs,then g,A and B have a unique common fixed point provided any one of the following hold

$M(A(u,u,\ldots\ldots u),B(v,v,\ldots\ldots\ldots v),t) << M(gu,gv,t)$ (3.5)

$0<\mu<\frac{s-1}{s^k-1}$ (3.6)

4. RESULT AND DISCUSION

Proof of theorem : As $v,u\in X$ s.t. $\lim_{n\to\infty}$ $y_n=v=g(u)=A(u,u,\ldots\ldots u)=B(u,u,\ldots\ldots\ldots u)$. Since the pairs (g,A) and (g,B) are weakly compatible pairs,we have

$ggu=g(A(u,u,\ldots.u)=A(gu,gu,\ldots\ldots gu)$

and $ggu=g(A(u,u,\ldots\ldots\ldots u)=A(gu,gu,\ldots\ldots\ldots\ldots gu)$

suppose (3.5) hold then we have

$M(gu,ggu,t)=M(A(u,u,\ldots.u),B(gu,gu,\ldots.,gu),t)<<M(gu,ggu,t)$ So $gu=ggu$

i.e. $gu=v$ is a common fixed point of g,A and B.If (3.6) hold ;by (3.2) for any k we have

$M(ggu,gu,t)=M(gB(u,u,\ldots\ldots u),B(u,u,\ldots\ldots u),t)=M(B(gu,gu,\ldots\ldots gu),B(u,u,\ldots.u),t)\preceq$
$sM(B(gu,gu,\ldots..gu),A(gu,gu,\ldots\ldots.gu),t)$
$+s^2M(A(gu,gu,\ldots.gu),B(gu,gu,\ldots..gu),t)+\ldots$
$+s^{k+1}M(A(gu,gu,\ldots gu),B(gu,gu,\ldots gu),t)$

$\preceq s\mu\Psi\{M(ggu,ggu,t),\ldots\ldots.M(ggu,ggu,t),\ldots..$

$M(ggu,ggu,t)+s^2\mu\Psi M(M(ggu,ggu,t),\ldots\ldots\ldots.$

$M(ggu.gu.t), M(gu,gu,t)+\ldots\ldots\ldots\ldots S^{k-1}\mu\Psi(M(ggu,gu,t),M(gu,gu,t),\ldots M(gu,gu,t)\preceq$
$(s+s^2+\ldots\ldots\ldots\ldots+s^{k-1})\,\mu M(ggu,gu,t)$

$\preceq (1+s+s^2+\ldots\ldots\ldots\ldots+s^{k-1})\,\mu M(ggu,gu,t)$

$\preceq \frac{s^k-1}{s-1}\,\mu M(ggu,gu,t)$

5. CONCLUSION

Hence $M(ggu,gu,t)=1$

So $ggu=gu$ and $gv=v$. Therefore g,A and B have common fixed point p.

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