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Contraction Theorem In Fuzzy 2-Metric Space

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Abstract: *The aim of present paper is to obtain a common fixed point theorem in fuzzy 2-metric space. My purpose is generalize the several known results.*

Keywords and Phrases: *fuzzy 2-metric spaces, compatibility*

AMS subject Classification(2000): *54H25, 47H10.*

1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [9] in 1965, who introduced the concept of fuzzy set. Kramosil and Michalek [4] developed the fuzzy metric space and later George and Veeramani [1] modified the notion of fuzzy metric spaces by introducing the concept of continuous t – norm.. Jungck[3] introduced the notion of compatible mappings. However the study of common fixed points of noncompatible mappings is also equally interesting. R.P.Pant [5-6] initiated the study of common fixed points of noncompatible mappings satisfying contractive type conditions. While compatibility is useful mainly in the study of common fixed points of contractive type mapping pairs and often requires the assumption of continuity and completeness .

2. PRELIMINARIES

Definition(2.1):A binary operation : $[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm if $\{[0,1],*\}$ is an abelian topological monoid with unit 1 such that $a*b\le c*d$ whenever $a\le c$ and $b\le d$, $a,b,c,d\in[0,1]$

Definition(2.2):A **fuzzy 2-metric space** is a space X in which for each triple of points x,y,z there exits a real function $M(x,y,z,t)$ such that :

(i)to each pair of distinct points x,y,z in there exists a point t in [0,1] then $M(x,y,z,t)\neq 0$;

(ii) $M(x,y,z,t) = 1$, when at least two of x,y,z are equal;

(iii)$M(x,y,z,t)= M(y,z,x,t)= M(x,z,y,t)$;

(iv)$M(x,y,z,t_1+t_2+t_3)\ge M(x,y,w,t_1) * M(x,w,z,t_2) * M(w,y,z,t_3)$,for all w in X and t_1, t_2, t_3 in [0,1].It is easily seen that M is non - negative.

Definition(2.3):A sequence$\{x_n\}$in a fuzzy 2-metric space(X,M,$*$)is said to be **convergent** with lim x in X if $\lim_{n\to\infty} M(x_n,x,z,t)=1$,for all z in X and t>0.

Definition(2.4):A sequence $\{x_n\}$ in a fuzzy 2-metric space (X,M,$*$) is said to be a **Cauchy sequence** if $\lim_{m,n\to\infty} M(x_m,x_n,z,t)=1$,for all z in X and t>0.

Definition(2.5):A fuzzy 2-metric space (X,M,$*$)is said to be **complete 2-metric space** if every Cauchy sequence in X is convergent.

Definition(2.6):Two mappings A and B of a fuzzy 2-metric space (X,M,$*$) into itself are said to be **compatible** if $\lim_{n\to\infty} M(ABx_n,BAx_n,z,t)=1$ whenever $\{x_n\}$ is a sequence in X such that: $\lim_{n\to\infty} Ax_n=u$, and $\lim_{n\to\infty} Bx_n=u$, for some $u\in X$ and t>0.

Definition(2.7):Let S and T be mappings from a fuzzy 2-metric space (X,M,$*$) into itself,then S and T are said to be **compatible type (A)** if $\lim_{n\to\infty} M(STx_n,TTx_n,z,t)=1$, and $\lim_{n\to\infty} M(TSx_n,SSx_n,z,t)=1$,whenever $\{x_n\}$ is a sequence in X such that: $\lim_{n\to\infty} Sx_n= u$, and $\lim_{n\to\infty} Tx_n= u$, for some u in X

Definition(2.8): Let S and T be mappings from a fuzzy 2-metric space (X,M,$*$) into itself,then S and T are said to be **compatible type (P)** if $\lim_{n\to\infty} M(SSx_n,TTx_n,z,t)=1$, whenever

$\{x_n\}$ is a sequence in X such that: $\lim_{n\to\infty} Sx_n = u$, and $\lim_{n\to\infty} Tx_n = u$, for some u in X

3. MATERIAL AND METHOD

Theorem(3.1):Let (X,M,*) be a fuzzy 2-metric space and let A and B be continuous self mappings of X , then $\{AB^n x\}$ converges to Aa. If A,B satisfying the following condition M(ABx,ABy,z,t) ≥ M(Ax,Ay,z,t/α) s.t. α ∈ [0,1)

Then B has a fixed point a.

4. RESULT AND DISCUSSION

Proof of theorem: It can be proof in four Steps

Step 1: M($AB^{n+1}x, AB^n x$,z,t)=1

Since M($AB^{n+1}x, AB^n x$,z,t)≥ M(ABx, Ax ,z,t/ α^n)

Therefore M($AB^{n+1}x, AB^n x$,z,t) =1 as α ∈ [0,1)

Step 2: { $AB^n x$ } is bounded sequence

Suppose { $AB^n x$ } is unbounded then there exist $\{n(k)\}_{k=1}^{\infty}$ s.t.n(1)=0 and for each k ∈ N,n(k+1) is 'minimal'

So M($AB^{n(k+1)}x, AB^{n(k)}x$,z,t) <0 and M($Tg^m x, Tg^{n(k)}x$,z,t) ≥ 0,

for all m = n(k)+1,n(k)+2,……,n(k+1)+1

But M($AB^{n(k+1)}x, AB^{n(k)}x$,z,t) ≥M($AB^{n(k+1)}x, AB^{n(k+1)-1}x, z, t)$ *M($AB^{n(k+1)-1}x, AB^{n(k)}x, z, t)$

≥M($AB^{n(k+1)}x, AB^{n(k+1)-1}x, z, t)$ *0

So M($AB^{n(k+1)}x - AB^{n(k)}x$,z,t) $\to 0$ as k $\to \infty$

Therefore M($AB^{n(k+1)}x, AB^{n(k)}x, z, t)$ ≥M($AB^{n(k+1)-1}x, AB^{n(k)-1}x, z, t/\alpha)$ contradiction

Step 3: $\{AB^n x\}_{n=1}^{\infty}$ is a Cauchy sequence since M($AB^m x, AB^n x, z, t)$ ≥ M($AB^{m-n}x, Ax, z, t/\alpha^n$) for α ∈ [0,1)

Therefore M($AB^m x, AB^n x, z, t)$ =1

Step 4: B has a unique fixed point

5. CONCLUSION

As $\{AB^n x\}_{n=1}^{\infty}$ is a Cauchy sequence ,then $AB^n x$ =Aa.

So M($AB^{n+1}x, ABa, z, t)$ ≥ M($AB^n x, Aa$,z,t/α) →1 for α ∈ [0,1)

Therefore ABa=Aa or Ba=a ,hence B has a fixed point .

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REFERENCES

[1] George,A. and Veeramani, P:_On some results in fuzzy metric space, Fuzzy Sets and Systems 64 (1994) 395-399.

[2] Helpern, S.: Fuzzy mapping and fixed point theorem, J.Math. Anal. App. 83 (1981), 566-569.

[3] Jungck,G.: compatible amppings and common fixed points ,Int. J math sci 9 (1986) 771-779.

[4] Kramosil,I and Michalek,J,: Fuzzy metric and Statistical metric spaces, Kybernetica ,Vol. 11, p. 326 – 334, 1975.

[5] Pant, R.P.: R weakly commuting and common fixed points of non compatible maps, Ganita 49(1998)19-27.

[6] Pant,R.P.: Common fixed point theorems for contractive maps, J math anal. appl 226 (1998)251-258.

[7] Pathak, H.K., Chang, S.S. and Cho, Y.J.: Fixed point theorems for compatible mappings of type(P), Ind.Jour. of Math. 36(2) (1994),151-166.

[8] Sharma,Aradhana and Sao,G.S.:fixed point theorem in fuzzy2-metric space , IJACSM ISSN 2249-3042. Vol.5, No.2, Nov.2015, 145-148.

[9] Zadeh,L.A.:Fuzzy sets,Inform and control 8,(1965),338-353.