Reliability Assessment of A.T.M. with Redundant Central Unit

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Abstract: *In this paper, the authors have done their analysis to compute the working capability of a ATM system. In this study, the authors assume single ATM. The capability of the system is affected by all the units of the system. The authors have been used two parallel redundant central computers to improve system's overall performance. Single bank computer is available to fulfill user's requirements.*

Using supplementary variables technique, the author has done mathematical formulation of the system. This mathematical model has been solved by employing Laplace transform. Probability of states has been computed. Reliability of considered system has been obtained. Steady-state behavior of the system and a particular case has also been obtained to improve practical utility of the system.

An important reliability parameter, viz. mean time to failure of considered system has also been computed to improve practical utility of the model. Graphical illustration followed by a numerical computation has been appended at the end to highlight important results of the study.

Keywords: *Reliability assessment, working capability, supplementary variables, Laplace transform, steady-state behavior etc.*

1. INTRODUCTION

An automated teller machine (ATM) is a computerized tele-communication device that provides the customers of a financial institution with access to financial transactions in a public space without the need for a human clerk or bank teller. On most modern ATMs, the customer is identified by inserting a plastic ATM card with a magnetic stripe or a plastic smart card with a chip that contains a unique card number and some security information, such as an expiration date or CVC (CVV). Security is provided by the customer entering a personal identification number (PIN). They are sometimes referred to as "ATM machines", an example of RAS syndrome. Fig-1(a) shows an ATM mouth.

Fig 1(a): *Shows ATM mouth*

Using an ATM, customers can access their bank accounts in order to make cash withdrawals (or credit card cash advances) and check their account balances.

2. REVIEW & LITERATURE

British Actor Reg Varney: Use the world's first ATM in 1967 located at a branch of Barclays Bank, Enfield. The system was developed by De La Rue.

The first mechanical cash dispenser was developed and built by Luther George Simjian, installed in 1939 in New York city by the Bank of New York, but removed after 6 months due to the lack of customer acceptance.

Thereafter, the history of ATMs paused for over 25 years, until De La Rue developed the first electronic ATM, which was installed first in Enfield town in North London United Kingdome on 27th June 1967 by Barclays Bank. This instance of invention is credited to John Shepherd Barron although various other engineers were awarded patents for related technologies at the time. Shepherd Barron was awarded an OBE in the 2005 New York's Honors list.

The first person to use the machine was the British variety artist and actor Reg Vauney. The first ATMs accepted only a single use token or voucher which was retained by the machine. These worked on various principles including radiation and low coercivity magnetism that was wiped by the card reader to make fraud more difficult. The machine dispensed prepackaged envelopes containing ten Ponds sterling. The idea of a PIN stored on the card was developed by the British engineer James Good Fellow in 1965. Fig-1(b) represents an ATM station with two parallel machines

3. MATERIAL & METHODS

In this paper, the authors have done their analysis to compute the working capability of a ATM system [1,4,5]. In this study, the authors assume single ATM. The capability of the system is affected by all the units of the system. Block diagram of considered system has been shown in fig-1(c). The authors have been used two parallel redundant central computers to improve system's overall performance. Single bank computer is available to fulfill user's requirements. Fig-1(d) represents the transition state diagram for considered system.

Using supplementary variables technique, the authors have done mathematical formulation of the system. This mathematical model has been solved by employing Laplace transform. Probability of states (depicted in fig-1(d)), has been computed. Reliability of considered system has been obtained. Steady-state behavior of the system and a particular case has also been obtained to improve practical utility of the system.

An important reliability parameter, viz. mean time to failure of considered system has also been computed to improve practical utility of the model. Graphical illustration followed by a numerical computation has been appended at the end to highlight important results of the study.

Fig 1(b): *Represents an ATM station with two parallel machines*

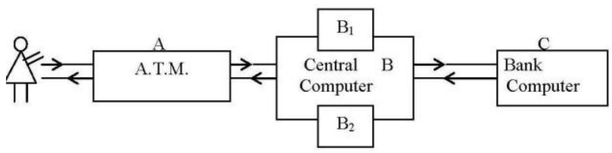

Fig 1(c): *System configuration*

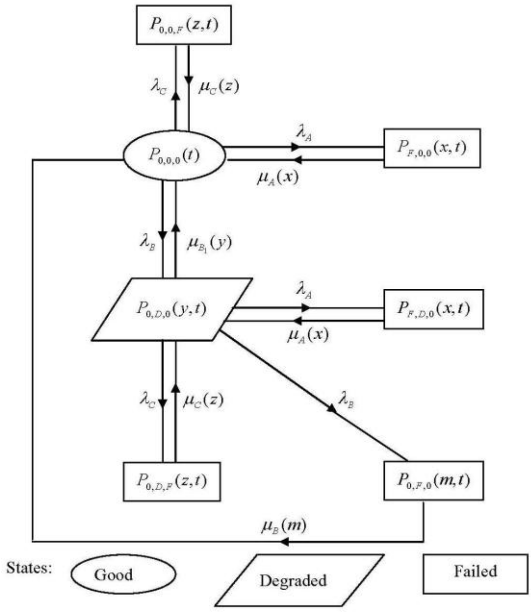

Fig 1(d): *State-transition diagram*

The following notations have been used throughout this study:

λ_i	:	Failure rate of subsystem $i(=A,B,C)$.
$\mu_i(j)\Delta$	:	The first order probability that i^{th} failure can be repaired in the time interval $(j, j+\Delta)$, conditioned that it was not repaired upto the time *j*.
$P_{0,0,0}(t)$	:	Pr {at time *t*, system is all operable as subsystems A, B and C are operable}.
$P_{F,0,0}(x,t)\Delta\, etc.$	:	Pr {at time *t*, system is failed due to failure of subsystem A}. Elapsed repair time lies in the interval $(x, x+\Delta)$.
$P_{0,D,0}(y,t)\Delta$	:	Pr {at time *t*, system is in degraded state due to failure of any one part of subsystem B}. Elapsed repair time lies in the interval $(y, y+\Delta)$.
$P_{F,D,0}(x,t)\Delta$ etc.	:	Pr {at time *t*, system is in failed state due to failure of subsystem A while one part of subsystem B has already failed}. Elapsed repair time for subsystem A lies in the interval $(x, x+\Delta)$.

$\bar{S}_i(j)$: $=\mu_i(j)\exp.\{-\int \mu_i(j)dj\}$, $\forall\ i \text{ and } j$

$D_i(j)$: $\frac{1-\bar{S}_i(j)}{j}$, $\forall\ i \text{ and } j$

M.T.T.F. : Mean time to failure

L.T. : Laplace transform

s : L.T. variable

The following assumptions have been associated with this model:

1. Initially, the whole system is good and operable.
2. All failures follow exponential time distribution and are S-independent.
3. All repairs follow general time distribution and are perfect.
4. There are two central computers working in parallel redundancy.
5. On failure of any one central computer, the whole system works in degraded state.
6. Repair facilities are always available and there is no time lap between a failure and start of repair.

4. RESULTS

4.1 Formulation of Mathematical Model

Probability considerations and limiting procedure yields the following set of difference-differential equations [4-5], which is continuous in time and discrete in space, governing the behavior of considered system:

$$\left[\frac{d}{dt}+\lambda_A+\lambda_B+\lambda_C\right]P_{0,0,0}(t)=\int_0^\infty P_{F,0,0}(x,t)\mu_A(x)dx+\int_0^\infty P_{0,D,0}(y,t)\mu_{B_1}(y)dy$$
$$+\int_0^\infty P_{0,0,F}(z,t)\mu_C(z)dz+\int_0^\infty P_{0,F,0}(m,t)\mu_B(m)dm \quad ...(1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right]P_{F,0,0}(x,t)=0 \quad ...(2)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_C+\mu_{B_1}(y)\right]P_{0,D,0}(y,t)=0 \quad ...(3)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_C(z)\right]P_{0,0,F}(z,t)=0 \quad ...(4)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_A(x)\right]P_{F,D,0}(x,t)=0 \quad ...(5)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\mu_B(m)\right]P_{0,F,0}(m,t)=0 \quad ...(6)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\mu_C(z)\right]P_{0,D,F}(z,t)=0 \quad ...(7)$$

Boundary conditions are:

$$P_{F,0,0}(0,t)=\lambda_A\ P_{0,0,0}(t) \quad ...(8)$$

$$P_{0,D,0}(0,t)=\lambda_B\ P_{0,0,0}(t)+\int_0^\infty P_{F,D,0}(x,t)\mu_A(x)dx+\int_0^\infty P_{0,D,F}(z,t)\mu_C(z)dz \quad ...(9)$$

$$P_{0,0,F}(0,t)=\lambda_C\ P_{0,0,0}(t) \quad ...(10)$$

$$P_{F,D,0}(0,t)=\lambda_A\ P_{0,D,0}(t) \quad ...(11)$$

$$P_{0,F,0}(0,t)=\lambda_B\ P_{0,D,0}(t) \quad ...(12)$$

$$P_{0,D,F}(0,t)=\lambda_C\ P_{0,D,0}(t) \quad ...(13)$$

Initial conditions are:

$P_{0,0,0}(0)=1$, otherwise all state probabilities are zero at $t=0$. ...(14)

4.2 Solution of the Model

Taking Laplace transforms of equations (1) through (13) subjected to initial conditions (14), we obtain:

$$[s+\lambda_A+\lambda_B+\lambda_C]\bar{P}_{0,0,0}(s)=1+\int_0^\infty \bar{P}_{F,0,0}(x,s)\mu_A(x)dx+\int_0^\infty \bar{P}_{0,D,0}(y,s)\mu_{B_1}(y)$$
$$+\int_0^\infty \bar{P}_{0,0,F}(z,s)\mu_C(z)dz+\int_0^\infty \bar{P}_{0,F,0}(m,s)\mu_B(m)dm \quad ...(15)$$

$$\left[\frac{\partial}{\partial x}+s+\mu_A(x)\right]\bar{P}_{F,0,0}(x,s)=0 \quad ...(16)$$

$$\left[\frac{\partial}{\partial y}+s+\lambda_A+\lambda_B+\lambda_C+\mu_{B_1}(y)\right]\bar{P}_{0,D,0}(y,s)=0 \quad ...(17)$$

$$\left[\frac{\partial}{\partial z}+s+\mu_C(z)\right]\bar{P}_{0,0,F}(z,s)=0 \quad ...(18)$$

$$\left[\frac{\partial}{\partial x}+s+\mu_A(x)\right]\bar{P}_{F,D,0}(x,s)=0 \quad ...(19)$$

$$\left[\frac{\partial}{\partial m}+s+\mu_B(m)\right]\bar{P}_{0,F,0}(m,s)=0 \quad ...(20)$$

$$\left[\frac{\partial}{\partial z}+s+\mu_C(z)\right]\bar{P}_{0,D,F}(z,s)=0 \quad ...(21)$$

$$\bar{P}_{F,0,0}(0,s)=\lambda_A\ \bar{P}_{0,0,0}(s) \quad ...(22)$$

$$\bar{P}_{0,D,0}(0,s)=\lambda_B\ \bar{P}_{0,0,0}(s)+\int_0^\infty \bar{P}_{F,D,0}(x,s)\mu_A(x)dx+\int_0^\infty \bar{P}_{0,D,F}(z,s)\mu_C(z)dz \quad ...(23)$$

$$\bar{P}_{0,0,F}(0,s)=\lambda_C\bar{P}_{0,0,0}(s) \quad ...(24)$$

$$\bar{P}_{F,D,0}(0,s)=\lambda_A\bar{P}_{0,D,0}(s) \quad ...(25)$$

$$\bar{P}_{0,F,0}(0,s)=\lambda_B\bar{P}_{0,D,0}(s) \quad ...(26)$$

$$\bar{P}_{0,D,F}(0,s)=\lambda_C\bar{P}_{0,D,0}(s) \quad ...(27)$$

Now integrating equation (16) with the help of boundary condition (22), we get

$$\bar{P}_{F,0,0}(x,s)=\lambda_A\ \bar{P}_{0,0,0}(s)\exp\{-sx-\int \mu_A(x)dx\}$$

Integrating this again w.r.t. x from 0 to ∞, we have

$$\bar{P}_{F,0,0}(s)=\lambda_A\ \bar{P}_{0,0,0}(s)\frac{1-\bar{S}_A(s)}{s}$$

or, $\bar{P}_{F,0,0}(s)=\lambda_A\ \bar{P}_{0,0,0}(s)D_A(s)$ ...(28)

Similarly, solving (18) subjected to (24), we get

$$\bar{P}_{0,0,F}(z,s)=\lambda_C\ \bar{P}_{0,0,0}(s)\exp\left\{-sz-\int\mu_C(z)dz\right\}$$

or, $\bar{P}_{0,0,F}(s)=\lambda_C\ \bar{P}_{0,0,0}(s)D_C(s)$ ...(29)

Again, integrate (19), (20), (21) with the help of conditions (25), (26) and (27) respectively, we have

$$\bar{P}_{F,D,0}(x,s)=\lambda_A\ \bar{P}_{0,D,0}(s)\exp\left\{-sx-\int\mu_A(x)dx\right\}$$

$$\Rightarrow \bar{P}_{F,D,0}(s)=\lambda_A\ \bar{P}_{0,D,0}(s)D_A(s) \quad ...(30)$$

$$\bar{P}_{0,F,0}(m,s)=\lambda_B\ \bar{P}_{0,D,0}(s)\exp\left\{-sm-\int\mu_B(m)dm\right\}$$

$$\Rightarrow \bar{P}_{0,F,0}(s)=\lambda_B\ \bar{P}_{0,D,0}(s)D_B(s) \quad ...(31)$$

and

$$\bar{P}_{0,D,F}(z,s)=\lambda_C\ \bar{P}_{0,D,0}(s)\exp\left\{-sz-\int\mu_C(z)dz\right\}$$

$$\Rightarrow \bar{P}_{0,D,F}(s)=\lambda_C\ \bar{P}_{0,D,0}(s)D_C(s) \quad ...(32)$$

Now, simplifying equation (23) with the help of (30) and (32), we have

$$\bar{P}_{0,D,0}(0,s)=\lambda_B\ \bar{P}_{0,0,0}(s)+\lambda_A\ \bar{P}_{0,D,0}(s)\bar{S}_A(s)+\lambda_C\ \bar{P}_{0,D,0}(s)\bar{S}_C(s) \quad ...(33)$$

Using (33), integrate (17), one can obtain

$$\bar{P}_{0,D,0}(y,s)=\bar{P}_{0,D,0}(0,s)\exp\left\{-(s+\lambda_A+\lambda_B+\lambda_C)y-\int\mu_{B_1}(y)dy\right\}$$

$$\Rightarrow \bar{P}_{0,D,0}(s)=\bar{P}_{0,D,0}(0,s)D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)$$

or,

$$\bar{P}_{0,D,0}(s)=\left\{\lambda_B\bar{P}_{0,0,0}(s)+\bar{P}_{0,D,0}(s)\left[\lambda_A\bar{S}_A(s)+\lambda_C\bar{S}_C(s)\right]\right\}D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)$$

$$\Rightarrow \bar{P}_{0,D,0}(s)=\frac{\lambda_B\bar{P}_{0,0,0}(s)D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)}{1-\left[\lambda_A\bar{S}_A(s)+\lambda_C\bar{S}_C(s)\right]D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)}$$

or $\bar{P}_{0,D,0}(s)=A(s)\ \bar{P}_{0,0,0}(s)$ (say) ...(34)

where

$$A(s)=\frac{\lambda_B D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)}{1-\left[\lambda_A\bar{S}_A(s)+\lambda_C\bar{S}_C(s)\right]D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)}$$

Finally, simplifying (15) subjected to relevant expressions, we obtain

$$\bar{P}_{0,0,0}(s)=\frac{1}{B(s)}$$

where

$$B(s)=s+\lambda_A+\lambda_B+\lambda_C-\lambda_A\bar{S}_A(s)+\lambda_C\bar{S}_C(s)-\lambda_B A(s)\bar{S}_B(s)$$
$$-\left[\lambda_B+\lambda_A A(s)\bar{S}_A(s)+\lambda_C A(s)\bar{S}_C(s)\right]\bar{S}_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)$$

Thus, we obtain the following L.T. of all the transition-state probabilities in terms of B(s):

$$\bar{P}_{0,0,0}(s)=\frac{1}{B(s)} \quad ...(35)$$

$$\bar{P}_{F,0,0}(s)=\frac{\lambda_A D_A(s)}{B(s)} \quad ...(36)$$

$$\bar{P}_{0,D,0}(s)=\frac{A(s)}{B(s)} \quad ...(37)$$

$$\bar{P}_{0,0,F}(s)=\frac{\lambda_C D_C(s)}{B(s)} \quad ...(38)$$

$$\bar{P}_{F,D,0}(s)=\frac{\lambda_A A(s)D_A(s)}{B(s)} \quad ...(39)$$

$$\bar{P}_{0,F,0}(s)=\frac{\lambda_B A(s)D_B(s)}{B(s)} \quad ...(40)$$

$$\bar{P}_{0,D,F}(s)=\frac{\lambda_C A(s)D_C(s)}{B(s)} \quad ...(41)$$

where

$$A(s)=\frac{\lambda_B D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)}{1-\left[\lambda_A\bar{S}_A(s)+\lambda_C\bar{S}_C(s)\right]D_{B_1}(s+\lambda_A+\lambda_B+\lambda_C)} \quad ...(42)$$

and

$$B(s)=s+\lambda_A+\lambda_B+\lambda_C-\lambda_A\bar{S}_A(s)-\lambda_C\bar{S}_C(s)-\lambda_B A(s)\bar{S}_B(s)$$
$$-\left[\lambda_B+\lambda_A A(s)\bar{S}_A(s)+\lambda_C A(s)\bar{S}_C(s)\right]\bar{S}_{B_1}(s+\lambda_A+\lambda_B+\lambda_C) \quad ...(43)$$

4.3 Steady-State Behavior of the System

Using final value theorem of L.T., viz., $\underset{t\to\infty}{Lim}\,P(t)=\underset{s\to 0}{Lim}\,s\ \bar{P}(s)=P$ (say), provided limit on left exits, we obtain the following steady-state behavior [1-3] of considered system from equations (35) through (41):

$$P_{0,0,0}=\frac{1}{B'(0)} \quad ...(44)$$

$$P_{F,0,0}=\frac{\lambda_A M_A}{B'(0)} \quad ...(45)$$

$$P_{0,D,0}=\frac{A(0)}{B'(0)} \quad ...(46)$$

$$P_{0,0,F}=\frac{\lambda_C M_C}{B'(0)} \quad ...(47)$$

$$P_{F,D,0}=\frac{\lambda_A A(0)M_A}{B'(0)} \quad ...(48)$$

$$P_{0,F,0}=\frac{\lambda_B A(0)M_B}{B'(0)} \quad ...(49)$$

$$P_{0,D,F}=\frac{\lambda_C A(0)M_C}{B'(0)} \quad ...(50)$$

where $M_i=-\bar{S}'_i(0)=$ Mean time to repair i^{th} failure

$$A(s)=\frac{\lambda_B D_{B_1}(\lambda_A+\lambda_B+\lambda_C)}{1-\left[\lambda_A+\lambda_C\right]D_{B_1}(\lambda_A+\lambda_B+\lambda_C)}$$

and $B'(0)=\left[\frac{d}{ds}B(s)\right]_{s=0}$

4.4 Particular Case

When all repairs follow exponential time distribution

In this case, setting $\bar{S}_i(j)=\mu_i/(j+\mu_i)$, $\forall$ i and j, in equations (35) through (41), we obtain the following L.T. of various transition-states [5] depicted in fig-1(b):

$$\bar{P}_{0,0,0}(s)=\frac{1}{E(s)} \quad ...(51)$$

ISSN 2455-4863 (Online) www.ijisset.org Volume: 2 Issue: 3 | March 2016

$$\overline{P}_{F,0,0}(s) = \frac{\lambda_A}{E(s).(s+\mu_A)} \quad ...(52)$$

$$\overline{P}_{0,D,0}(s) = \frac{F(s)}{E(s)} \quad ...(53)$$

$$\overline{P}_{0,0,F}(s) = \frac{\lambda_C}{E(s).(s+\mu_C)} \quad ...(54)$$

$$\overline{P}_{F,D,0}(s) = \frac{\lambda_A F(s)}{E(s).(s+\mu_A)} \quad ...(55)$$

$$\overline{P}_{0,F,0}(s) = \frac{\lambda_B F(s)}{E(s).(s+\mu_B)} \quad ...(56)$$

$$\overline{P}_{0,D,F}(s) = \frac{\lambda_C F(s)}{E(s).(s+\mu_C)} \quad ...(57)$$

where

$$F(s) = \frac{\lambda_B}{(s+\lambda_A+\lambda_B+\lambda_C+\mu_{B_1}) - \left[\frac{\lambda_A\mu_A}{s+\mu_A} + \frac{\lambda_C\mu_C}{s+\mu_C}\right]} \quad ...(58)$$

and

$$E(s) = s+\lambda_A+\lambda_B+\lambda_C - \frac{\lambda_A\mu_A}{s+\mu_A} - \frac{\lambda_C\mu_C}{s+\mu_C} - \frac{\lambda_B F(s)\mu_B}{s+\mu_B}$$

$$-\left[\lambda_B + F(s)\left(\frac{\lambda_A\mu_A}{s+\mu_A} + \frac{\lambda_C\mu_C}{s+\mu_C}\right)\right]\frac{\mu_{B_1}}{s+\lambda_A+\lambda_B+\lambda_C+\mu_{B_1}} \quad ...(59)$$

4.5 Reliability and M.T.T.F. Evaluation

From equation (35), we have

$$\overline{R}(s) = \frac{1}{s+\lambda_A+\lambda_B+\lambda_C}$$

Taking inverse L. T., we get

$$R(t) = \exp\{-(\lambda_A+\lambda_B+\lambda_C)t\} \quad ...(60)$$

Also, $M.T.T.F. = \int_0^\infty R(t)dt$

$$= \frac{1}{\lambda_A+\lambda_B+\lambda_C} \quad ...(61)$$

4.6 Numerical Illustration

For a numerical illustration, let us consider the values

$\lambda_A = 0.006$, $\lambda_B = 0.003$, $\lambda_C = 0.001$, and t = 0,1,2,-----10.

Table 1: *computes reliability at different time points*

t	R(t)
0	1
1	0.99005
2	0.980199
3	0.970446
4	0.960789
5	0.951229
6	0.941765
7	0.932394
8	0.923116
9	0.913931
10	0.904837

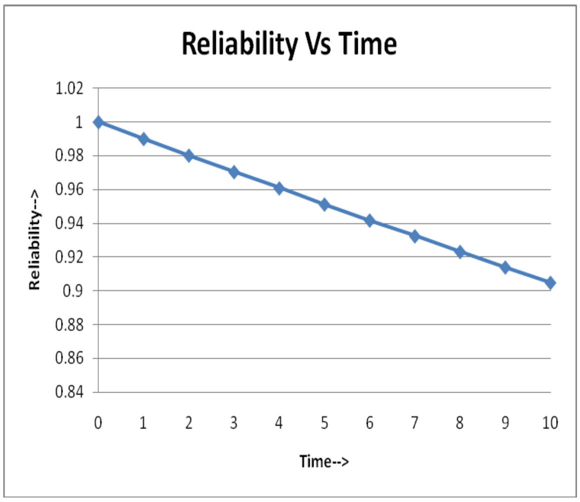

Fig 2: *represents reliability vs time*

Table 2: *computes MTTF for different failure rates*

λ_B	M.T.T.F.
0	142.8571
0.001	125
0.002	111.1111
0.003	100
0.004	90.90909
0.005	83.33333
0.006	76.92308
0.007	71.42857
0.008	66.66667
0.009	62.5
0.01	58.82353

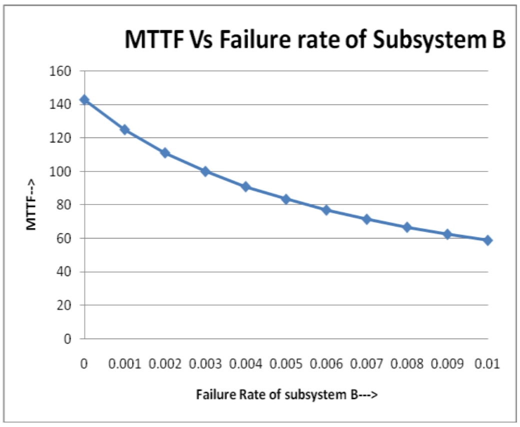

Fig 3: *represents MTTF vs failure rate of subsystem B*

5. DISCUSSION

Using above values in equations (60) and (61), we obtain the tables (1) and (2), respectively. The corresponding graphs have been shown through figure (2) and (3) respectively. These graphs illustrate the variation into obtained results.

6. SUMMARY AND CONCLUSION

We have given the reliability of considered system, for various values of time t, in table-1. The graph of this has been shown in fig-2. Critical examination of fig-2 reveals that reliability of the system decreases in a constant manner approximately. It should be noted that there are no sudden jumps in the values of reliability of considered system.

Table-2 gives the values of M.T.T.F. for various *values of failure rate of subsystem B.* It's graph has been sketched in fig-3. In this graph, we observe that M.T.T.F. decreases catastrophically in the beginning but thereafter it decreases in a constant manner. In earlier study [4] no care was given to central computer although it is very important component of the system. The results obtain in this study are much better than earlier study. We found that, the reliability and MTTF of the system remains higher than in [5].

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