Fixed Point Theorem of Type (P) in Fuzzy 2-Metric Space

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Abstract: *The aim of present paper is to obtain a common fixed point theorem in fuzzy 2-metric space. my purpose here is to generalize the several known results.*

Keywords and Phrases: *fuzzy 2-metric spaces, compatibility*

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1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [9] in 1965, who introduced the concept of fuzzy set. Kramosil and Michalek [4] developed the fuzzy metric space and later George and Veeramani [1] modified the notion of fuzzy metric spaces by introducing the concept of continuous t–norm.. Jungck [3] introduced the notion of compatible mappings. However the study of common fixed points of noncompatible mappings is also equally interesting. R.P.Pant [5-6] initiated the study of common fixed points of noncompatible mappings satisfying contractive type conditions. While compatibility is useful mainly in the study of common fixed points of contractive type mapping pairs and often requires the assumption of continuity and completeness.

2. PRELIMINARIES

Definition(2.1):A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $\{[0,1],*\}$ is an abelian topological monoid with unit 1 such that $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, $a,b,c,d \in [0,1]$

Definition (2.2): A fuzzy 2-metric space is a space X in which for each triple of points x,y,z there exits a real function $M(x,y,z,t)$ such that :

(i) to each pair of distinct points x,y,z in there exists a point t in [0,1]then $M(x,y,z,t) \neq 0$;

(ii) $M(x,y,z,t) = 1$, when at least two of x,y,z are equal;

(iii) $M(x,y,z,t) = M(y,z,x,t) = M(x,z,y,t)$

(iv) $M(x,y,z,t_1+t_2+t_3) \geq M(x,y,w,t_1)*M(x,w,z,t_2)*M(w,y,z,t_3)$, for all w in X and t_1, t_2, t_3 in [0,1].It is easily seen that M is non-negative.

Definition(2.3): A sequence $\{x_n\}$ in a fuzzy 2-metric space $(X,M,*)$ is said to be **convergent** with lim x in X if $\lim_{n\to\infty} M(x_n,x,z,t)=1$, for all z in X and $t>0$.

Definition (2.4): A sequence $\{x_n\}$ in a fuzzy 2-metric space $(X,M,*)$ is said to be a **Cauchy sequence** if $\lim_{m,n\to\infty} M(x_m,x_n,z,t)=1$, fo all z in X and $t>0$.

Definition (2.5): A fuzzy 2-metric space $(X,M,*)$ is said to be **complete** if every Cauchy sequence in X is convergent.

Definition (2.6): Two mappings A and B of a fuzzy 2-metric space $(X,M,*)$ into itself are said to be **compatible** if $\lim_{n\to\infty} M(ABx_n,BAx_n,z,t)=1$ whenever $\{x_n\}$ is a sequence in X such that : $\lim_{n\to\infty} Ax_n=u$, and $\lim_{n\to\infty} Bx_n= u$, for some $u \in X$ and $t>0$.

Definition (2.7): Let S and T be mappings from a fuzzy 2-metric space $(X,M,*)$ into itself,then S and T are said to be **compatible type(A)** if $\lim_{n\to\infty} M(STx_n,TTx_n,z,t)=1$,

and $\lim_{n\to\infty} M(TSx_n,SSx_n,z,t)=1$,whenever$\{x_n\}$ is a sequence in X such that : $\lim_{n\to\infty} Sx_n= u$, and $\lim_{n\to\infty} Tx_n= u$, for some u in X

Definition(2.8): Let S and T be mappings from a fuzzy 2-metric space $(X,M,*)$ into itself, then S and T are said to be **compatible type (P)** if $\lim_{n\to\infty} M(SSx_n,TTx_n,z,t)=1$,whenever $\{x_n\}$ is a sequence in X such that : $\lim_{n\to\infty} Sx_n=u$,

and $\lim_{n\to\infty} Tx_n = u$, for some u in X

3. MATERIAL AND METHOD

Theorem(3.1):Let $(X,M,*)$ be a fuzzy 2-metric space and let A and B be continuous self mappings of X,S and T be a continuous multivalued mapping from X into $CB(X)$ satisfying the following conditions:

$A(X) \subset T(X)$ and $B(X) \subset S(X)$ (3.1.1)

$M(Ax,By,z,t) \geq \min \{M(Sx,Ty,z,t),M(Ax,Sx,z,t),$ $M(By,Ty,z,t),M(Ax,Ty,z,t),M(By,Sx,z,t)\}(3.1.2)$

For any $x,y,z \in X$ and $t>0$,if$\{A,T\}$ and $\{B,S\}$ are compatible mappings of type(P),there exist point $u \in X$,so u is a coincidence point of A,B,S and T.

4. RESULT AND DISCUSSION

Proof of theorem: Let x_0 be any arbitrary point in X. Define a sequence $\{x_n\}$ in X by

$r_{2n} = Tx_{2n+1} = Ax_{2n}$ $r_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$

then we have $M(Ax_{2n},Bx_{2n+1},z,t)$

$= \min M(r_{2n},r_{2n+1},z,t) \geq \min \{M(r_{2n-1},r_{2n},z,t),$

$\qquad M(r_{2n},r_{2n-1},z,t),M(r_{2n+1},r_{2n},z,t),$

$\qquad M(r_{2n},r_{2n},z,t),M(r_{2n+1},r_{2n-1},z,t)\}$

$\qquad\qquad \ldots\ldots\ldots\ldots$

$\Rightarrow M(Ax_{2n},Bx_{2n+1},z,t) \geq M(Ax_{2n-1},Bx_{2n},z,t)$

But $\{Ax_n\}$ and $\{Bx_{n+1}\}$ are Cauchy sequences in X .Therefore

$\lim_{n\to\infty} M(Ax_{2n-1},Bx_{2n},z,t)=1$

Hence $\lim_{n\to\infty} M(Ax_{2n},Bx_{2n+1},z,t)=1.$

Also $1 = \lim_{n\to\infty} M(AAx_{2n},TTx_{2n+1},z,t)$

$\qquad = \lim_{n\to\infty} M(Ar_{2n},Tr_{2n},z,t)=M(Au,Tu,z,t)$

i.e. $Au=Tu$

And $1 = \lim_{n\to\infty} M(BBx_{2n+1},SSx_{2n+2},z,t)$

$\qquad = \lim_{n\to\infty} M(Br_{2n+1},Sr_{2n+1},z,t)=M(Bu,Su,z,t)$

So that $Bu = Su$,hence $Au=Bu=Tu=Su$

5. CONCLUSION

Therefore u is a coincidence point of A,B,S,T.

Similarly

$\lim_{n\to\infty} M(AAx_n,BBx_n,z,t) = 1$

$\qquad\qquad = \lim_{n\to\infty} M(ABx_n,BBx_n,z,t)$

Therefore A and B are compatible mappings of type (A) and type(P).

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