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Common Fixed Point and Compability Mapping in Fuzzy 2-Metric Space

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Abstract: *The aim of present paper is to obtain a fixed point by contraction and compability in fuzzy 2-metric space of Kannan type .Our purpose is to generalize the several known results.*

Keywords and Phrases: *fuzzy 2-metric spaces, contraction mapping, compability, fixed point.*

AMS Subject Classification (2000): *54H25, 47H10.*

1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [14] in 1965, who introduced the concept of fuzzy set. George and Veeramani [3] modified the notion of fuzzy metric spaces by introducing the concept of continuous t – norm. The fuzzy metric space was introduced by Kramosil and Michalek [5],Khan M.S, Pathak H.K. & George R.[6] compatible mappings of Type (A−1) and Type A−2 in fuzzy metric spaces, Sao,G.S. and Shrivastava,U.K. [11] introduce the notion of compability of type(D).

2. PRELIMINARIES

The following definitions and results will be needed in the sequel.

DEFINITION 2.1([13]) Let X be any set. A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition(2.2)[3]:Abinary operation : $[0,1]x[0,1]\rightarrow[0,1]$ is a continuous t-norm if {[0,1],*} is an abelian topological monoid with unit 1 such that $a*b\le c*d$ whenever a $\le c$ and $b\le d$, $a,b,c,d\in[0,1]$

Definition(2.3)[3]:A **fuzzy 2-metric space** is a space X in which for each triplet of points x,y,z there exits a real function M(x,y,z,t) such that :

(i) to each pair of distinct points x,y,z in X there exists a point t in[0,1] s.t. $M(x,y,z,t)\neq0$;

(ii) M(x,y,z,t) = 1, when x,y,z are equal;

(iii) M(x,y,z,t)= M(y,x,z,t) ;

(iv) $M(x,y,z,t_1+t_2+t_3)$

$\geq M(x,y,z,t_1)*M(x,y,z,t_2)*M(x,y,z,t_3)$,

for all x,y,z in X and t_1,t_2,t_3 in [0,1].It is easily seen that M is non -negative.

Definition(2.4)[3]:A sequence$\{x_n\}$in a fuzzy 2-metric space(X,M,*)is said to be **convergent** with lim x in X if

$$\lim_{n\to\infty} M(x_n,x,z,t)=1$$

for all x,y,z inX and t>0.

Definition(2.5)[3]:A sequence $\{x_n\}$ in a fuzzy 2-metric space (X,M,*) is said to be a **Cauchy sequence** if

$$\lim_{m,n\to\infty} M(x_m,x_n,z,t)=1$$

for all x,y,z in X and t>0.

Definition(2.5)[3]:A fuzzy 2-metric space (X,M,*)is said to be **complete** fuzzy 2-**metric space** if every Cauchy sequence in X is convergent.

DEFINITION 2.6([1]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,*) are said to be compatible if,

$\lim_{n\to\infty} M(ASx_n, SAx_n,z, t) = 1$ whenever $\{x_n\}$ is a sequence such that:

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.7([4]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,*) are said to be weakly compatible if, they commute at coincidence points.That is,Ax=

Sx implies that AS*x*=SA*x* for all *x* in X.

DEFINITION 2.8([2]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,*) are said to be subcompatible if, $\lim_{n\to\infty} M(ASx_n, SAx_n,z,t) = 1$ whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.9([7]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,∗) are said to be reciprocally continuous if,

$$\lim_{n\to\infty} M(ASx_n, Ax,z, t) = 1$$

and $\lim_{n\to\infty} M(SAx_n, Sx,z, t) = 1$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

It is noted that if A and S are both continuous, they are obviously reciprocally continuous but the converse need not be true. For this, we have the following example:

EXAMPLE 1.1([7]) Consider two mappings A and S defined on X = [2, 20] with the usual Euclidean metric d, defined by the rule $A_2 = 2$, $Ax = 3$ if $x > 2$ and $S_2 = 2$, $Sx = 6$ if $x > 2$. Then, A and S are reciprocally continuous but are not continuous.

DEFINITION 2.10([6]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,∗) are said to be compability of type(A) if,

$$\lim_{n\to\infty} M(ASx_n, SSx_n ,z, t) = 1$$

and $\lim_{n\to\infty} M(SAx_n, AAx_n ,z, t) = 1$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

3. MATERIAL AND METHOD

Theorem(3.1):Let (X,M,∗) be a fuzzy 2-metric space and let T,S:X→X are continuous mappings of X, then $\{Tx_n\}$and $\{Sx_n\}$ converges to p.If T and S satisfying the following condition:

$M(Tx,Sy,z,t)\le aM(x,Tx,z,t)$

$+bM(Sy.Tx,z,t)+cM(x,Sy,z,t)$

$+dM(x,y,z,t)$

When x,y,z in X and $t\in[0,1]$s.t. $a+b+2c+d<1$,then T and S has a common fixed point p.

4. RESULT AND DISCUSSION

Proof of theorem:Consider

$Q_{n-1} = M(x_n,x_{n+1},z,t)$ if $x=x_n$, $y=x_{n+1}$ then we have

$Q_{n+1}\le aQ_n+bQ_{n+1}+c(Q_n+Q_{n+1})+dQ_n$

$\Rightarrow (1-b-c)Q_{n+1} \le (a+c+d)\, Q_n$

$$\Rightarrow Q_{n+1} \le \frac{a+c+d}{1-b-c}\, Q_n$$

$$\Rightarrow Q_{n+1} \le k\, Q_n \text{ when } k=\frac{a+c+d}{1-b-c}<1,$$

Hence $a+b+2c+d<1$

$\Rightarrow Q_n \le k\, Q_{n-1}$

$\Rightarrow Q_{n+1} \le k^2\, Q_{n-1}$

$\Rightarrow Q_n \le k^n\, Q_0$

But $Q_0=M(x_0,x_1,z,t)$ is a Cauchy fuzzy 2-metric space .So Q_n be Cauchy fuzzy metric space .Also we can easily prove that it is convergent.

$M(Tx_n,p,z,t)\le M(Tx_n,p,z,t/2)*$
$M(Tx_n,p,z,t/4)*M(Tx_n,p,z,t/8)*\ldots\ldots$

$M(Tx_n,p,z,t/2^n)$

$\Rightarrow M(Tx_n,p,z,t)\to 1$

Therefore it converge at the point p .

Also we can easily show that point p is unique.

Also we can show that this is a **type(A)**,Since

$M(TSx,SSx,z,t)\le aM(Sx,TSx,z,t)$

$+bM(SSx.TSx,z,t)+cM(Sx,SSx,z,t)$

$+dM(Tx,TSx,z,t)$

s.t. x,y,z in X and $t\in[0,1]$ with $a+b+2c+d<1$

Consider $Q_{n+1} = M(x_{n+4},x_{n+3},z,t)$ if $x=x_n$,$Sx_n=x_{n+2}$ and $Tx_n=x_{n+1}$ then we have

$M(TSx_n,SSx_n,z,t)\le aM(Sx_n,TSx_n,z,t)$

$+bM(SSx_n.TSx_n,z,t)+cM(Sx_n,SSx_n,z,t)$

$+dM(Tx_n,TSx_n,z,t)$

s.t. x,y,z in X and $t\in[0,1]$

Hence after substituting values,we have

$(1-b-c)Q_{n+1}=(a+c+d)Q_n$

$\Rightarrow Q_{n+1} \leq \frac{a+c+d}{1-b-c} Q_n$

$\Rightarrow Q_{n+1} \leq k\, Q_n$ when $k=\frac{a+c+d}{1-b-c}<1$,

s.t. x,y,z in X and $t \in [0,1]$ with $a+b+2c+d<1$

As $\{x_n\}$ is a sequence with $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

So we have prove this is a compability of type(A).

5. CONCLUSION

As Q_n or $\{Tx_n\}_{n=1}^{\infty}$ and $\{Sx_n\}_{n=1}^{\infty}$ are Cauchy sequence ,then $Tx_n=p=Sx_n$

So $M(Tx_n,p,z,t) \to 1$

$\Rightarrow M(TSx_n,SSx_n,z,t) \to 1$

Therefore T and S have a common fixed point p as well as T and S are compability of type(A)

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