Wave Field Modeling in Layered Medium with Combined Hierarchical Inclusions and Method of Interpretation

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Abstract: Monitoring studies using variable electromagnetic and acoustic fields are currently being conducted as part of active observation systems with a source of corresponding excitation. The present work is devoted to the description of algorithms for solving the direct problem for acoustic excitation in the framework of the model of composite hierarchical structures with various physical and mechanical properties. To solve the inverse problem, a new method has been developed for determining the contours of hierarchical structures within the framework of the same model. The method is based on the use of an explicit solution to the theoretical inverse problem. An iterative process is developed for solving the theoretical inverse problem for the case of determining the configurations of 2D hierarchical inclusions of the l-th, m-th, and s-th ranks located one above the other in different layers of the N-layer medium for acoustic monitoring. The degree of filling with inclusions of each rank for all three hierarchical inclusions is different. When interpreting the results of monitoring, it is necessary to use data from observation systems that can be configured to study the hierarchical structure of the medium (these are acoustic (in the dynamic version) and electromagnetic active monitoring systems). According to the data of electromagnetic monitoring, the morphology of the distribution of disintegration zones in the orts of the Tashtagol mine, which are prone to stability disturbance phenomena, was revealed. Acoustic dynamic monitoring systems are more preferable for studying the state of the array from the point of view of migration and the dynamics of the intensity of hierarchical zones. The use of these algorithms is necessary for analyzing the dynamic state of a rock massif of deep-seated deposits, as well as for studying the state of an oil-saturated reservoir to increase oil recovery.

Keyword: electromagnetic¹, acoustic², theory³, processing⁴, visualization⁵, methods⁶, examples⁷, combined⁸, hierarchical⁹, inclusions¹⁰, iterative¹¹, algorithms¹², direct¹³, inverse¹⁴, solution¹⁵, equation¹⁶, theoretical¹⁷, inverse¹⁸, problem¹⁹.

1. INTRODUCTION

The most important result of geomechanical and geodynamic studies of the past century was the discovery of a close relationship between global geodynamic and local geomechanical processes caused by mining operations, especially in tectonically active zones. No less important result of the research was the conclusion about the fundamental role of the block-hierarchical structure of rocks and massifs for explaining the existence of a wide range of nonlinear geomechanical effects and the emergence of complex self-organizing geosystems. Hierarchical structure is typical for many systems, especially for the Earth's lithosphere, where more than 30 hierarchical levels from tectonic plates with a length of thousands of kilometers to individual mill grains of millimeter size were identified by geophysical studies [1]. Thus, the earth's crust is not a continuous medium, but a discrete block system and, like any synergetic discrete ensemble, has hierarchical and self-similar properties [2].

Possible reduction in economic indicators in the global oil and gas industry is due to outdated and largely erroneous concepts on the structure of non-oil and gas fields in the geological section. We can note several negative consequences in the economy of the fuel and energy complex, caused by outdated and erroneous ideas. First, unjustifiably large volumes of drilling wells in those sections of the fields where either there are no deposits, or there are very poor deposits, the development of which is knowingly unprofitable. Secondly, the various methods of influencing the reservoir with the aim of increasing oil recovery in those areas where there is practically no oil only worsen the economic indicators with erroneous ideas about the genesis and spatial structure of the deposits. It is known that the volumes of injected liquid grow several times faster than the volumes of incremental oil production. Thirdly, unreasonably high expenses for drilling and pumping water not only reduce the overall profitability of hydrocarbon production, but also significantly increase the level of environmental pollution, the liquidation of which will require
additional costs. Fourth, the erroneous concepts of the origin and evolution of hydrocarbon deposits have led to such production technologies, under which there is a very rapid depletion of the deposits and significantly reduced the renewability of resources. All this unjustifiably quickly reduces economic indicators in developed fields. The main elements of the new paradigm in the fuel and energy complex are the search, development and technology of development of small-sized, but extraordinarily rich in reserves of active oil and gas generation. The geosoliton concept of the formation of hydrocarbons and the formation of large-scale, but rich in reservoirs, is the main conceptual basis of the new paradigm [3]. To date, new effective technologies can be attributed the results published in the paper [4].

The processes of development of oil and gas fields are associated with the motion of multiphase multi component media that are characterized by no equilibrium and non-linear rheological properties. The real behavior of reservoir systems is determined by the complexity of the rheology of moving fluids and the morphological structure of the porous medium, as well as by the variety of processes of interaction between the liquid and the porous medium [5]. Accounting for these factors is necessary for a meaningful description of filtration processes due to the nonlinearity, non-equilibrium and heterogeneity inherent in real systems. In this case, new synergic effects are revealed (loss of stability with the appearance of oscillations, the formation of ordered structures). This allows us to propose new methods for monitoring and managing complex natural systems that are tuned to account for these phenomena. Thus, the reservoir system from which to extract oil is a complex dynamic hierarchical system.

When constructing a mathematical model of a real object, it is necessary to use, as a priori information, active and passive monitoring data obtained during the current operation of the facility. In papers of [6,7], modeling algorithms were constructed for 3-D heterogeneities in electromagnetic case, for 2-D heterogeneities in the seismic case and for an arbitrary type of excitation source of an N-layer medium with hierarchical elastic inclusions located in the J-th layer. In the paper of [8], a new 2D modeling algorithm for sound diffraction on elastic and porous, fluid-saturated inclusions of hierarchical structure located in the J-th layer of an N-layer elastic medium was developed. In [9], modeling algorithms were constructed in the acoustic case for a 2-D heterogeneity and an arbitrary type of excitation source of an N-layer medium with a separate hierarchical anomaly density, stresses and plastic inclusions located in the J-th layer.

In this paper, using the method described in [6-9], an algorithm for modeling the acoustic field (longitudinal acoustic wave) has been developed in the form of an iterative process for solving a direct problem for the case of three hierarchical inclusions of l, m, s-ranks using 2D integral and integro-differential equations. The degree of hierarchy of inclusions is determined by the values of their ranks, which can be different. Hierarchical inclusions are located in different layers above each other: the top is anomalously stressed, the second is by fluid saturated and the third is anomaly dense (Figure 1).

![Figure 1](image)

Figure 1. The scheme of composite anomalously stressed (upper), fluid-saturated (medium) and anomaly dense (lower) heterogeneities of hierarchical type located in an N-layer elastic medium.

### 2.1 Algorithm of Modeling of Sound Diffraction on 2D Composite Heterogeneity of Hierarchical Type, Located in N-Layered Elastic Environment

An algorithm for modeling of sound diffraction on a two-dimensional elastic hierarchical inclusions located in the J-th layer of an N-layer medium is described in [7,9]. $G_{Sp,j}(M, M^0)$ - the source function of the seismic field, the boundary value problem for which was formulated in [10,11]. $k_{ji}^2 = \omega^2(\sigma_{ji} / \lambda_{ji})$ is the wave number for the longitudinal wave; in the above expression, the index $ji$ denotes the property of the medium inside the heterogeneity, $ja$ denotes the property of the medium outside the heterogeneity, $\lambda$ is the Lamé constant, $\sigma$ - density of medium, $\omega$ - circular frequency, $\bar{u} = \nabla \varphi$ - vector of displacements, $\varphi$ - potential of normal seismic field in layered medium in
the absence of heterogeneity: \( \varphi_{ij}^0 = \varphi_{ji}^0 \), \( \varphi_{ij}^0 \) - potential of a normal seismic field in a layered medium in the absence of a heterogeneity of the previous rank \( l \), if \( l = 2 \ldots L \), if \( l = 1 \), which coincides with the corresponding expression [9]. We assume that the density of the first hierarchical inclusion for all ranks \( j \) and the enclosing layer are identical, and elastic parameters of hierarchical inclusion for all ranks differ from the elastic parameters of the enclosing environment. Let the rank values for all hierarchical inclusions will be: \( l = m = s = 1 \), then the system of equations describing the propagation of the longitudinal acoustic wave in the first inclusion will be rewritten as:

\[
\frac{\kappa^2_{ij(j-1)} - \kappa^2_{ij(j-1)}}{2\pi} \int_{S_{3C3}} \varphi_{ij}(M)G_{Sp_{ij}(j-1)}(M, M^0) d\tau_M + \\
\varphi_{ij}^0(M^0) = \varphi_{ij}^0(M^0), M^0 \in S_{3C3}
\]

(1)

We calculate \( \varphi_{ij}(M^0), M^0 \in \Pi_{j-1} \) in the layer where the second hierarchical elastic inclusion is located, using expression (2), then the normal acoustic field potential for the second inclusion is written in the form:

\[
\varphi_{m-1}^0(M^0) = \varphi_{j}^0(M^0), M^0 \in \Pi_{j-1}
\]

The system of equations for the second elastic hierarchical inclusion of rank \( m = 1 \) has the form according to [6,7]:

\[
\frac{\kappa^2_{ijm(j-1)m} - \kappa^2_{ijm(j-1)m}}{2\pi} \int_{S_{2Cm}} \varphi_{ijm}(M)G_{Sp_{ijm}(j-1)m}(M, M^0) d\tau_M + \\
\varphi_{ijm}^0(M^0) = \varphi_{ijm}^0(M^0), M^0 \in S_{2Cm}
\]

(2)

We calculate \( \varphi_{ijm}(M^0), M^0 \in \Pi_{j-1} \) in the layer where the third hierarchical anomalous density inclusion is located using expression (4), then the normal potential of the acoustic field is:

\[
\varphi_{j}^0(M^0) = \varphi_{j}^0(M^0), M^0 \in \Pi_{j-1}
\]

We assume that the elastic parameters of the third hierarchical inclusion for all ranks \( s \) and the enclosing layer are the same, and the density of the hierarchical inclusion for all ranks differs from the density of the host environment, then the system of equations for the third hierarchical inclusion of rank \( s = 1 \) has the form according to [9]:

\[
\frac{\kappa^2_{ij(j-1)s} - \kappa^2_{ij(j-1)s}}{2\pi} \int_{S_{3C3}} \varphi_{ij}(M)G_{Sp_{ij}(j-1)s}(M, M^0) d\tau_M + \\
\varphi_{ij}^0(M^0) = \varphi_{ij}^0(M^0), M^0 \in S_{3C3}
\]

(5)

For the second elastic hierarchical inclusion of rank \( m = 2 \), we have:

\[
\frac{\kappa^2_{ijm(j-1)m} - \kappa^2_{ijm(j-1)m}}{2\pi} \int_{S_{2Cm}} \varphi_{ijm}(M)G_{Sp_{ijm}(j-1)m}(M, M^0) d\tau_M + \\
\varphi_{ijm}^0(M^0) = \varphi_{ijm}^0(M^0), M^0 \in S_{2Cm}
\]

(3)

We calculate \( \varphi_{ijm}(M^0), M^0 \in \Pi_{j-1} \) in the layer where the fourth hierarchical elastic inclusion is located, using expression (6), then the normal potential of the acoustic field is:

\[
\varphi_{j}^0(M^0) = \varphi_{j}^0(M^0), M^0 \in \Pi_{j-1}
\]

We assume that the elastic parameters of the fourth hierarchical inclusion for all ranks \( s \) and the enclosing layer are the same, and the density of the hierarchical inclusion for all ranks differs from the density of the host environment, then the system of equations for the fourth hierarchical inclusion of rank \( s = 1 \) has the form according to [9]:

\[
\frac{\kappa^2_{ij(j-1)s} - \kappa^2_{ij(j-1)s}}{2\pi} \int_{S_{3C3}} \varphi_{ij}(M)G_{Sp_{ij}(j-1)s}(M, M^0) d\tau_M + \\
\varphi_{ij}^0(M^0) = \varphi_{ij}^0(M^0), M^0 \in S_{3C3}
\]

(6)

The values of \( L, M, s \) and \( S \) are the maximum values of the ranks of the hierarchy for the three inclusions. In this paper \( L = 3, M = 3, s = 4, S = 4 \).

(6')

Let: \( l = l + 1; m = m + 1; s = s + 1 \). If \( l < 3 \) or \( l = 3 \), then we turn to the algorithm (1)-(6). If \( l > 3 \), and \( m = 2 \), then we...
calculate \( \varphi_j(M^0), M^0 \notin S_{3C_3} \in \Pi_j \) in the layer \( j \) using expression (7):

\[
\sigma_{j,i}(k_{j,i}^2 - k_{j,j}^2) \left\{ \frac{\varphi_j(M) G_{Sp,j}(M, M^0) d \tau_M + \varphi_{j-1}(M^0) - \varphi_{j-1}(M^0)}{S_{3C_3}} \right\} - \frac{(\sigma_{ja} - \varphi_{ja})}{\sigma_{j,m}^2} \int G_{Sp,j}(M, M^0) \frac{\varphi_{j,m}^2}{cn} dc = \varphi_j(M^0), M^0 \notin S_{3C_3} \in \Pi_j; \quad (7)
\]

\( \varphi_{m-1}(M^0) = \varphi_{m-1}(M^0), M^0 \in \Pi_j \) and we proceed to the algorithm (3) - (6), if \( m = 3 \), 
\( \varphi_{m-1}(M^0) = \varphi_{m-1}(M^0), M^0 \in \Pi_j \) then we turn to the algorithm (3) - (6). If \( m = 3 \) and \( s < 4 \) or \( s = 4 \), 
\( \varphi_{m,j}^s(M^0) = \varphi_{m,j}^s(M^0), M^0 \in \Pi_j \), then we go to (5) - (6). If \( s > 4 \), then we pass to (7).

\[
(k_{j,m}^2 - k_{j,j}^2) \frac{\varphi_j(M) G_{Sp,j}(M, M^0) d \tau_M + \varphi_{j,m}^0}{S_{2C_m}} - \frac{(\sigma_{ja} - \varphi_{ja})}{\sigma_{j,m}^2} \int G_{Sp,j} \frac{\varphi_{j,m}^2}{cn} dc = \varphi_j(M^0), M^0 \notin S_{2C_m}; \quad (7)
\]

where \( \alpha=1-\chi - \frac{K}{K_0} \), \( K = \lambda \cdot \text{complete compression module, } \chi \cdot \text{porosity, } K_0 \cdot \text{the true modulus of phase compressibility, pore hydrostatic pressure } p_2 \). The first equation of system (3) is rewritten in the form in accordance with the fact that in the second inclusion the heterogeneities of the second rank become fluid or oil-saturated. Then go to (5) - (6).

\[
\sigma_{j,i}(k_{j,i}^2 - k_{j,j}^2) \left\{ \frac{\varphi_j(M) G_{Sp,j}(M, M^0) d \tau_M + \varphi_{j-1}(M^0) - \varphi_{j-1}(M^0)}{S_{3C_3}} \right\} - \frac{(\sigma_{ja} - \varphi_{ja})}{\sigma_{j,m}^2} \int G_{Sp,j} \frac{\varphi_{j,m}^2}{cn} dc = \varphi_j(M^0), M^0 \notin S_{3C_3} \in \Pi_j; \quad (8)
\]

We calculate \( \varphi_j(M^0), M^0 \notin S_{3C_3} \in \Pi_j \) in all the layers \( j = 1, \ldots, N \) with the use of the expression (8). The algorithm stops if the hierarchy ranks become larger than the given numbers (6). If at some hierarchical level the structure of the local heterogeneity breaks down into several heterogeneities, then the double and contour integrals in the expressions (1-7) are taken over all heterogeneities of a given rank.

### 2.2 DISCUSSION

Iterative modeling algorithms are constructed in the seismic case in the acoustic approximation for a composite hierarchical heterogeneity. When constructing a complex seismic gravity model without taking into account the anomalous influence of a stress-strain state within a hierarchical inclusion, called the cushion of the deposit, an analysis of the anomalous acoustic effect using data on the propagation of a longitudinal wave shows that the influence of anomalous elastic parameters in the seismic model cannot be neglected, since they affect on the values of the unknown anomalous densities. If these values are used in constructing a density gravitational model without taking into account the effect of elastic parameters, these density values will not reflect the material composition of the analyzed medium. When constructing an anomalously strained geomechanical model without taking into account the anomalous influence of density heterogeneities inside the hierarchical inclusion, which is the substrate for a two-phase deposit, the values of the unknown anomalous elastic parameters that cause the anomalous stress state in the cushion using data on propagation of the transverse wave will be determined incorrectly. These values of elastic parameters will not reflect the real stress state of the analyzed medium over the fluid containing the deposit, which in turn is represented by a hierarchical multi-neighborhood environment. For the first time, the proposed iterative algorithm for modeling a hierarchically complex two-phase medium can be used to manage the production of viscous oil in mine conditions and light oil in sub horizontal wells [12]. The benefits of the new paradigm, of the new methods of oil and gas mapping and monitoring using geophysical wave fields on the base of developed mathematical algorithms can be seriously enlarged.

A new approach to the interpretation of wave fields has been developed to determine the contours or surfaces of composite local hierarchical objects. An iterative process has been developed to solve a theoretical inverse problem for the case of determining the configurations of 2D hierarchical inclusions of the 1-th, m-th, and s-th ranks located one above the other in different layers of the N-layer medium and various physical and mechanical properties for active acoustic monitoring with sources of longitudinal and transversal waves. When interpreting the results of monitoring, it is necessary to use data from such observation systems that can be configured to study the hierarchical structure of the environment. Such systems include acoustic (in the dynamic version) and electromagnetic monitoring systems. The hierarchical structure of the geological environment is clearly visible when analyzing rock samples taken from ore mines. On the other hand, the more complex the
environment, the each wave field introduces its information about its internal structure, therefore, the interpretation of the seismic and electromagnetic fields must be conducted separately, without mixing these databases. This result is contained in the explicit form of the equations of the theoretical inverse problem for a 2D electromagnetic field (E and H polarization), as well as for the propagation of a linearly polarized elastic wave when excited by an N-layer conducting or elastic medium with a hierarchical conducting or elastic inclusion located in the ν-th layer. In the present work, the inverse problem for a complicated hierarchical model of inclusions is considered. It can be used when conducting monitoring seismic and acoustic borehole studies to monitor the fluid return of oil fields, to analyze the dynamic state of a mountain range of deep-seated deposits which are under various mechanical effects.

3. 1 DEFINING THE CONTOURS OF COMBINED HIERARCHICAL INCLUSIONS LOCATED INTO A BLOCK LAYERED MEDIUM BY WAVE’S DATA OF ACTIVE ACOUSTIC AND ELECTROMAGNETIC MONITORING

An important role for understanding the formation and development of a hierarchy of structural levels of deformation in solids is played by theoretical and experimental results obtained on samples [13]. With their help, an approach has been developed that uses ideas about dissipative structures in nonequilibrium systems [14], for which self-organization processes take place at each of the hierarchical levels. As shown in [15], self-organization occurs when there is a hierarchical structure. This approach can be applied to the study of such natural and man-made systems, such as rock massifs, which are in the process of mining. The model of an open dynamic system [16] is applicable for their description. Analysis of the manifestations of self-organization processes can give an idea of the stability of the system and contribute to the development of criteria for the stability of the state of the array as a whole regarding the dynamic phenomena of a given energy class. This satisfies the statement expressed in [17], which consists in the hypothesis about the divisibility of medium scales. While the destruction of smaller scales fits into the concept of a nonstationary random process, for which the prediction of individual events is not possible.

In [18,19], using 3D electromagnetic induction space-time monitoring, it was possible to show that the model of a hierarchical discrete environment is applicable to describe the structure of an array of rocks of different material composition. Within the framework of a specific modification of the method, it was possible to trace two hierarchical levels. The zones of disintegration [22-23] in the near-working space are located asymmetrically in the soil and roof, which may be evidence of a non-equilibrium state of the system. These zones are located discretely, i.e. there are intervals of their total absence in the near-working space. The maximum changes in the massif, which is under the anthropogenic influence, manifest themselves in the change over time of the morphology of the spatial position of these zones (Figure 2 (a-d))
At present, theoretical results on modeling the electromagnetic and seismic field in a layered medium with inclusions of a hierarchical structure are in demand. Simulation algorithms are constructed in the electromagnetic case for the 3D heterogeneity, in the seismic case for the 2D heterogeneity [24-30]. It is shown, that with the increase of hierarchy degree of the environment, the degree of spatial distribution nonlinearity of the seismic and electromagnetic fields components increases also. That corresponds to the detailed monitoring experiments conducted in the hazardous mines of the Tashtagol mine and SUBR. The theory developed demonstrated how complex the process of integrating methods using an electromagnetic and seismic field to study the response of a medium with a hierarchical structure. This problem is inextricably linked with the formulation and solution of the inverse problem for the propagation of electromagnetic and seismic fields in such complex environments. In [31-33], the problem of constructing an algorithm for solving an inverse problem using the equation of a theoretical inverse problem for the 2D Helmholtz equation was considered. Explicit equations of the theoretical inverse problem are written for cases of electromagnetic field scattering (E and H polarization) and scattering of a linearly polarized elastic wave in a layered conducting and elastic medium with a hierarchical conducting or elastic inclusion, which are the basis for determining the contours of misaligned inclusions of the l-th rank of a hierarchical structure. Obviously, when solving the inverse problem, it is necessary to use observation systems set up to study the hierarchical structure of the environment as the initial monitoring data. In [38], a modeling algorithm was constructed for the acoustic monitoring data of a hierarchical two-phase geological environment with different physical and mechanical properties. In this paper, we construct an algorithm for reconstructing the contours of hierarchical composite structures associated with disintegration zones according to active acoustic monitoring using a source of longitudinal waves.

### 3.2 ALGORITHM FOR SOLUTION OF THE INVERSE PROBLEM OF 2-D SOUND DIFFRACTION IN N-LAYERED MEDIUM WITH COMPOSITE HIERARCHICAL INCLUSIONS.

The inverse problem of the diffraction of a linearly polarized elastic shear wave by a two-dimensional elastic heterogeneity of a hierarchical type located in a layer ν of an N-layered medium was solved in [33]. Here we consider this problem for a source of a longitudinal wave in the framework of a complicated model: the anomalously stressed hierarchical heterogeneity of the l-th rank will be located in the layer ν-1, the maximum value of the l-th rank is L, the initial value of the l-th rank is ll = 1; the anomalously elastic hierarchical heterogeneity of the m-th rank is located in the layer ν, the maximum value of the m-th rank is M, the initial value of the m-th rank is mm = 1, and the anomaly dense hierarchical heterogeneity of the s-th rank is ss = 1. We consider the algorithm for recovering 2D surfaces of hierarchical heterogeneities in the case when L < M < S. Let us write the equation of the theoretical inverse problem [33] for the scalar Helmholtz equation, to which our problem for the layer ν-1 reduces:

\[
2\pi U^+(M_0) = \iota((U^+_{\nu-1}(M) + U^+_{\nu-1}(M))(\frac{\partial G^l(M,M_0)}{\partial l} - \frac{\partial G(M,M_0)}{\partial n}) - \frac{h_{\nu-1}}{b_\nu} \frac{\partial U^+_{\nu-1}}{\partial n} + \\
+ \frac{\partial U^+_{\nu-1}}{\partial n}(\frac{1}{b_\nu})G^l(M,M_0) - (\frac{1}{b_\nu})G(M,M_0))dl; \quad (9)
\]

By this: \( h_{\nu-1} = \xi_{\nu-1}; b_\nu = \xi_\nu; b_{\nu-1} = \xi_{\nu-1}; l = ll \) \( (10) \)
\( \xi_{v-1} = \xi_{v} = \xi_{v+1}, \rho_{v-1}, \rho_{v}, \rho_{v+1} \) - the values of the elastic parameter Lamé \( \lambda \) and the density in the \( (v-1), (v) \) -th layer, in the layer where point \( M_0 \) is located inside the heterogeneity in the layer, with:

\[ \xi_{a(v)-1} = \lambda_{a(v)-1}, \xi_{a} = \lambda_{a}, \xi_{a+1} = \lambda_{a+1}; \rho_{a} = \rho_{a(v)-1}; \]

\[ U^+ = \phi^+_v; U_{v-1} = \phi^+_v; U_{v-1}^1 = \phi^+_v. \] (11)

Where \( \vec{u} = \text{grad} \phi \), if \( l = 1 \), then \( \phi^+_v \) is equal to the potential by absence of heterogeneities in the layered medium; if \( l > 1 \), then \( \phi^+_v \) (see the value calculated by formula (29))

\[ G(M, M_0) = G_{SP}(M, M_0); G^*(M, M) = G_{SP}^*(M, M); \]

\[ \kappa^2_{In(v)-1} = \omega^2 \frac{\rho_{v}(v)-1}{\rho_{v}(v)}, \kappa^2_{v-1} = \omega^2 \frac{\rho_{v}(v)-1}{\rho_{v}(v)}; k^2_{0} = \omega^2 \frac{\rho_{v}(v)-1}{\rho_{v}(v)}; \] (12)

The algorithm for calculating the Green function is written in [10-11]. Thus, the equation of the theoretical inverse problem is written in the form:

\[ 2\pi \phi^{v-1}(M_0) = \int \frac{G^*(M, M_0)}{\rho_{v}(v)} \left( \frac{\partial G^*(M, M_0)}{\partial n} \right) \xi_{v-1} d\Gamma + \sum_{k=0}^{1} \int \frac{G^*(M, M_0)}{\rho_{v}(v)} \left( \frac{\partial G^*(M, M_0)}{\partial n} \right) \xi_{v} d\Gamma \] (13)

Solving equation (13) with respect to the function describing the contour, we calculate the functions:

\[ \phi^+_v; \phi^+_v; \phi^+_v, \] by the algorithm for solving the direct problem [30] inside and outside the heterogeneity placed in a layered medium.

\[ \frac{k^2_{In(v)-1} - k^2_{v-1}}{2\pi} \xi_{v-1} \frac{\partial G^*(M, M_0)}{\partial n} d\Gamma + \frac{\rho_{v}(v)}{2\pi} \xi_{v-1} \frac{\partial G^*(M, M_0)}{\partial n} d\Gamma + \frac{\rho_{v}(v)}{2\pi} \xi_{v} \frac{\partial G^*(M, M_0)}{\partial n} d\Gamma + \frac{\rho_{v}(v)}{2\pi} \xi_{v} \frac{\partial G^*(M, M_0)}{\partial n} d\Gamma \] (14)

\[ \frac{k^2_{In(v)-1} - k^2_{v-1}}{2\pi} \xi_{v-1} \frac{\partial G^*(M, M_0)}{\partial n} d\Gamma + \frac{\rho_{v}(v)}{2\pi} \xi_{v} \frac{\partial G^*(M, M_0)}{\partial n} d\Gamma + \frac{\rho_{v}(v)}{2\pi} \xi_{v} \frac{\partial G^*(M, M_0)}{\partial n} d\Gamma \] (15)

Thus, the equation of the theoretical inverse problem is written in the form:

\[ 2\pi \phi^{v+(v)_0}(M_0) = \int \frac{G_{SP}(M, M_0)}{\rho_{v}(v)} \left( \frac{\partial G_{SP}(M, M_0)}{\partial n} \right) \xi_{v+1} d\Gamma + \sum_{k=0}^{1} \int \frac{G_{SP}(M, M_0)}{\rho_{v}(v)} \left( \frac{\partial G_{SP}(M, M_0)}{\partial n} \right) \xi_{v} d\Gamma \] (16)

\[ \frac{b_{v}}{b_{l}} \xi_{v}, \xi_{a}, \xi_{a+1}, \xi_{v}, \xi_{v+1}, \xi_{v-1}, \xi_{v+1} - \text{values of the elastic parameter Lamé and density in the } (v), (v+1), \text{layer, in the layer where the point } M_0 \text{ is located and inside the heterogeneity in the layer } v, \text{by that:} \]

\[ \xi_{a}, \xi_{a+1}, \xi_{a+1}, \xi_{v}, \xi_{v+1}, \xi_{v+1} - \text{values of the elastic parameter Lamé and density in the } (v), (v+1), \text{layer, in the layer where the point } M_0 \text{ is located and inside the heterogeneity in the layer } v, \text{by that:} \]

\[ \xi_{a}, \xi_{a+1}, \xi_{a+1}, \xi_{v}, \xi_{v+1}, \xi_{v+1} - \text{values of the elastic parameter Lamé and density in the } (v), (v+1), \text{layer, in the layer where the point } M_0 \text{ is located and inside the heterogeneity in the layer } v, \text{by that:} \]

\[ \xi_{a}, \xi_{a+1}, \xi_{a+1}, \xi_{v}, \xi_{v+1}, \xi_{v+1} - \text{values of the elastic parameter Lamé and density in the } (v), (v+1), \text{layer, in the layer where the point } M_0 \text{ is located and inside the heterogeneity in the layer } v, \text{by that:} \]
\[
(k_{1m}^2 - k_{2m}^2) \frac{2\pi}{2S_0} \int \frac{\partial \phi_m(M)G_{SP,v}(M,M')d\tau_M}{\rho_{m0}} + \frac{\rho_{m0}^2}{\rho_{m0}} \phi_m(M')d\tau_M = \frac{2\pi}{2S_0} \int \frac{\partial \phi_{m1}(M')d\tau_M}{\rho_{m0}} \phi_m(M')d\tau_M - 2\pi \phi_{m1}(M')d\tau_M
\]

- \left( \frac{\rho_{m0}^2}{\rho_{m0}} \right) \int \frac{\partial \phi_m(M)G_{SP,v}(M,M')d\tau_M}{\rho_{m0}} + \frac{\rho_{m0}^2}{\rho_{m0}} \phi_m(M')d\tau_M

\]

\[
\frac{\rho_{m0}}{\rho_{m0}} \int \frac{\partial \phi_m(M)G_{SP,v}(M,M')d\tau_M}{\rho_{m0}} + \frac{\rho_{m0}^2}{\rho_{m0}} \phi_m(M')d\tau_M
\]

\[
\phi_m(M'), M' \not\in S_{2Cm};
\]

\[
\phi_m(M), M \not\in S_{2Cm}, \in \Pi_{v+1};
\]

We write the equation of the inverse problem [33] for the scalar Helmholtz equation, to which our problem reduces, for the layer v+1, s = ss:

\[
2\pi \Phi_0 = \frac{1}{2\pi} \int \frac{\partial G^s(M,M)}{\partial n} d\tau_M - \frac{1}{2\pi} \int \frac{\partial G^s(M,M)}{\partial n} d\tau_M
\]

By:

\[
b_{v+1} = \xi_{v+1}; b_i = \xi_i; b_{d(v+1)s} = \xi_{d(v+1)s};
\]

\[
\xi_v, \xi_i, \xi_{d(v+1)s}, \rho, \rho_i, \rho_{d(v+1)s}; \text{ values of the elastic parameter}
\]

Lame \( \lambda \) and density in the (v+1) -th layer, in the layer where point \( M_0 \) is located and inside the heterogeneity in the layer (v+1):

\[
\xi_{d(v+1)s} = \lambda_{v+1}; \xi_{v+1} = \lambda_{v+1}; \xi_i = \lambda_i;
\]

\[
\rho_i \neq \rho_{d(v+1)s};
\]

\[
U = \phi^v; U_{v+1} = \phi_{v+1}; U_{v+1} = \phi_{(v+1)m}
\]

\[
G(M,M_0) = G_{SP}(M,M_0); G^s(M,M) = G_{SP}^s(M,M); \partial D, dL
\]

\[
k_{d(v+1)s}^2 = \omega^2 \rho_{d(v+1)s} = k_{d(v+1)s}^2 = \omega^2 \rho_{d(v+1)s} = k_{d(v+1)s}^2 = \omega^2 \rho_{d(v+1)s} = k_{d(v+1)s}^2
\]

Thus, the equation of the inverse theoretical problem is written in the form:

\[
\frac{\rho_{m0}^2}{\rho_{m0}} \phi_m(M')d\tau_M - 2\pi \phi_{m1}(M')d\tau_M = \frac{2\pi}{2S_0} \int \frac{\partial \phi_{m1}(M')d\tau_M}{\rho_{m0}} \phi_m(M')d\tau_M - 2\pi \phi_{m1}(M')d\tau_M
\]

Solving the equation (27) with respect to the function \( \tau_s(\phi) \) describing the contour \( \partial D_\phi \), we calculate the functions: \( \phi_{v+1}; \phi_{v+1}(v); \phi_{v+1} \) by the algorithm for solving the direct problem [30] inside and outside the heterogeneity placed in a layered medium.

\[
\frac{k_{1m}^2 - k_{2m}^2}{2\pi} \frac{2\pi}{2S_0} \int \frac{\partial \phi_m(M)G_{SP,v}(M,M')d\tau_M}{\rho_{m0}^2} + \frac{\rho_{m0}^2}{\rho_{m0}} \phi_m(M')d\tau_M
\]

\[
\phi_m(M), M \not\in S_{2Cm}, \in \Pi_{v+1};
\]

If the rank \( 1 \leq L \), \( \phi_{(-1)}(M_0) \) is calculated by the formula (29):

\[
\frac{\rho_{m0}^2}{\rho_{m0}^2} \phi_{m1}(M_0) = \frac{2\pi}{2S_0} \int \frac{\partial \phi_{m0}(M)G_{SP,v}(M,M_0)d\tau_M}{\rho_{m0}^2} + \frac{\rho_{m0}^2}{\rho_{m0}^2} \phi_{m1}(M_0)
\]

\[
\phi_{m0}(M), M \not\in S_{3Cm}, \in \Pi_{v+1};
\]

\[
\phi_{m0}(M), M \not\in S_{3Cm}, \in \Pi_{v+1};
\]

\[
mm = mm + 1 \text{ and we go to (16). If the rank is } m > M, \text{ then } \phi_{v+1}(M_0) \text{ is calculated by the formula (31)}:
\]

\[
\frac{\rho_{m0}^2}{\rho_{m0}^2} \phi_{m1}(M_0) = \frac{2\pi}{2S_0} \int \frac{\partial \phi_{m0}(M)G_{SP,v}(M,M_0)d\tau_M}{\rho_{m0}^2} + \frac{\rho_{m0}^2}{\rho_{m0}^2} \phi_{m1}(M_0)
\]

\[
\phi_{m0}(M), M \not\in S_{3Cm}, \in \Pi_{v+1};
\]
ss = ss + 1 and we go to (23). If the rank is S, then $\varphi_{(v+1)}(M_0)$ is calculated by the formula (32) in all layers: $v = 1, ..., N$.

$$\rho_{(v)0}(k_{(v)0}^2 - k_{(v)}^2) \frac{\|\varphi_s(M)G_{SP,(v)}(M,M^0)d\tau_s + + \varphi_{s-1}(M^0)}{\rho(M^0)^{2\pi} S_{3CS}} \int G_{SP,(v)}(v) \frac{\partial \varphi_s}{\partial n} dc =$$

$$= \varphi_{s(v)}(M^0), M^0 \not\in S_{3CS} \in \Pi_v; \quad (32)$$

The result is the defining of all the nested surfaces of hierarchical heterogeneities according to the ranks of their hierarchy and given physical and mechanical properties.

4. CONCLUSIONS

In this paper, using the method described in [6–9], an algorithm for modeling the acoustic field (longitudinal acoustic wave) has been developed in the form of an iterative process for solving a direct problem for the case of three hierarchical inclusions of $l, m, s$-ranks using 2D integral and integro-differential equations. The degree of hierarchy of inclusions is determined by the values of their ranks, which can be different. Hierarchical inclusions are located in different layers above each other: the top is anomalously stressed, the second is by fluid saturated and the third is anomaly dense.

When solving the inverse problem, it is necessary to use observation systems set up to study the hierarchical structure of the environment as basic monitoring data. On the other hand, the more complex the environment, the more wave field introduces its information about its internal structure, therefore, the interpretation of the seismic and electromagnetic fields must be conducted separately, without mixing these databases. From the constructed theory it follows that with an increase in the degree of hierarchy of the environment, the degree of spatial nonlinearity of the distribution of the components of the seismic and electromagnetic fields increases, which indicates the impossibility of using the methods of linearizing the task when creating interpretation methods.

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