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Fixed Point in Cone-Fuzzy Metric Space

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Abstract: *Partial metric spaces were introduced by Matthews[10] in 1994,on the other hand ,in 1989 the concept of b-metric spaces was introduced by Bakhtin[1] as a generalization of metrices. In this paper ,cone b-metrices are introduced as a generalization of metric space .Now we improve the result for cone b-fuzzy metric spaces.*

Keywords: *Common fixed points, fuzzy metric space, compatible maps,*

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1. INTRODUCTION

The concept of Fuzzy sets was initially investigated by Zadeh [11] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. In this paper we deal with the Fuzzy metric space defined by Kramosil and Michalek [9] and modified by George and Veeramani [4]. Recently, Grabiec [5] has proved fixed point results for Fuzzy metric space. For the sake of completeness, we recall some definitions and known results in Fuzzy metric space.

2. PRELIMINARIES

Definition(2.1):Abinary operation : $[0,1]x[0,1] \to [0,1]$ is a continuous t-norm if $\{[0,1],*\}$ is an abelian topological monoid with unit 1 such that $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, $a,b,c,d \in [0,1]$

Definition(2.2):A **fuzzy 2-metric space** is a space X in which for each triplet of points x,y,z there exits a real function M(x,y,z,t) such that :

(i)to each pair of distinct points x,y,z in X there exists a point $t \in [0,1]$ then $M(x,y,z,t) \neq 0$;

(ii)M(x,y,z,t)=1,when x,y,z are equal;

(iii)M(x,y,z,t)= M(y,x,z,t) ;

(iv)$M(x,y,z,t_1+t_2+t_3) \geq M(x,y,z,t_1)*M(x,y,z,t_2)*M(x,y,z,t_3)$,

for all x,y,z in X and $t_1,t_2,t_3 \in [0,1]$.It is easily seen that M is non -negative.

Definition(2.3):A sequence$\{x_n\}$in a fuzzy 2-metric space(X,M,∗)is said to be **convergent** with lim $x \in X$ if

$$\lim_{n\to\infty} M(x_n,x,z,t)=1, \text{for all } x,y,z \in X \text{ and } t>0.$$

Definition(2.4):A sequence $\{x_n\}$ in a fuzzy 2-metric space (X,M,∗) is said to be a **Cauchy sequence** if

$$\lim_{m,n\to\infty} M(x_m,x_n,z,t)=1, \text{for all } x,y,z \in X \text{ and } t>0.$$

Definition(2.5):A fuzzy 2-metric space (X,M,∗)is said to be **complete** fuzzy 2-**metric space** if every Cauchy sequence in X is convergent.

DEFINITION (2.6):If a non empty set X and s≥1, then a function $d:X^2 \to R^+$ is a b-metric on X if for all $x,y,z \in X$ satisfies

(1) d(x,y)=0 iff x=y;

(2) d(x,y)=d(y,x) ;

(3) $d(x,z) \leq s[d(x,y)+d(y,z)]$

then (X,d) is b-metric space

DEFINITION (2.7): If a non empty set X and a function $p:X^2 \to R^+$ is a partial-metric on X if for all $x,y,z \in X$ satisfies

(1) x=y iff p(x,x)=p(x,y)=p(y,y);

(2) $p(x,x) \leq p(x,y)$;

(3) p(x,y)=p(y,x);

(4) $p(x,y) \leq p(x,z)+p(z,y)-p(z,z)$

then (X,p) is partial-metric space

DEFINITION(2.8):If a non empty set X and a function $N:X^3 \to R^+$ is a N –cone metric on X if for all $x,y,z,a \in X$ satisfies

(1) $0\le N(x,x,x)$;

(2) $N(x,y,z)=0$ iff $x=y=z$;

(3) $N(x,y,z)\le N(x,x,a)+N(y,y,a)+N(z,z,a)$

then (X,N) is cone-metric space

DEFINITION(2.9):If a non empty set X and $s \le 1$, then a function $M:X^3\to[0,1]$ is a M –cone fuzzy metric on X if for all $x,y,u,v,t \in X$ and $x\ne u$,$y\ne v$ satisfies

(1) $0\le M(x,y,t)$; $M(x,y,t)=1$ iff $x=y$

(2) $M(x,y,t)=M(y,x,t)$

(3) $M(x,y,t) \ge s[M(x,u,t)*M(u,v,t)*M(v,y,t)]$

then $(X,M,*)$ is cone-fuzzy metric space

3. MATERIAL AND METHOD

Theorem(3.1):Let $(X,M,*)$ is a M cone fuzzy metric space and let

$M(Tx,Ty,t)\ge \lambda M(x,y,t)$

Then T has a fixed point

4. RESULT AND DISCUSSION

Proof : Consider $Tx_n = x_{n+1}$ for any $x\in X$,the iterative sequence $\{T^n x\}$

$n\in N$ converges to fixed point if $x_n\ne x_{n+1}$ for all $n\ge 0$

setting $Qn=M(x_n,x_{n+1},t)$ then

$M(x_n,x_{n+1},t)=M(Tx_{n-1},Tx_n,t)\ge \lambda M(x_{n-1},x_n,t)$ by Banach contraction

Then $Q_n \ge \lambda Q_{n-1}>Q_{n-1}$

$Q_n \ge \lambda^2 Q_{n-2}$

$Q_n \ge \lambda^n Q_{n-n}$

But $Q_0=M(x_0,x_1,t)\ge M(x_0,x_1,t/2)* M(x_0,x_1,t/4)* M(x_0,x_1,t/8)$………

$\to 1$

Similarly $M(x_n,x_{n+m},t)\ge s[M(x_n,x_{n+1},t)* M(x_{n+1},x_{n+2},t)* M(x_{n+2},x_{n+m},t)$

$\ge s[Q_n+Q_{n+1}]*s^2[M(x_{n+2},x_{n+3},t)*M(x_{n+3},x_{n+4},t)*M(x_{n+4},x_{n+m},t)]$

$\ge s[Q_n+Q_{n+1}]*s^2[Q_{n+2}+Q_{n+3}]*s^3[Q_{n+4}+Q_{n+5}]*$……

$\ge 1*1*1*1$………………………………

$\to 1$ as $s\le 1$

Therefore $\{x_n\}$is a Cauchy sequence and X is complete then we can easily shows that $M(x_n,u,t) \to 1$,then $\lim_{n\to\infty} x_n = u$

5. CONCLUSION

$\{x_n\}$is Cauchy sequence as well as convergent ,so limit be unique

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