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Common Fixed Point and Compability in Fuzzy 2-Metric Space

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Abstract: *Fuzzy metric space is first defined by Kramosil and Michalek in 1975. Many authors modified Fuzzy metric space and proved fixed point results in Fuzzy metric space. Singh B. and Chauhan were first introduced the concept of compatible mappings of Fuzzy metric space and proved the common fixed point theorem in 2000.Cho et were introduced the concept of compatible mapping of type (P). In this paper, we obtain a fixed point by compability in fuzzy 2-metric space .Our purpose is generalize the several known results.*

Keywords: *Common fixed points, fuzzy metric space, compatible maps,*

2000 AMS Subject Classification: *Primary 47H10, Secondary 54H25*

1. INTRODUCTION

The concept of Fuzzy sets was initially investigated by Zadeh [17] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. In this paper we deal with the Fuzzy metric space defined by Kramosil and Michalek [8] and modified by George and Veeramani [3]. Recently, Grabiec [4] has proved fixed point results for Fuzzy metric space. In the sequel, Singh and Chauhan [15]introduced the concept of compatible mappings of Fuzzy metric space and proved the common fixed point theorem. Pathak, Chang and Cho [9] introduced the concept of compatible mapping of type (P),Pathak and Khan[11]introduced the concept of compatible mapping of type (B), Pathak, Cho,Kang and Madharia[12] introduced the concept of compatible mapping of type (C),Sao and Srivastava[16] introduced the concept of compatible mapping of type (D). For the sake of completeness, we recall some definitionsand knownresults in Fuzzy metric space.

2. PRELIMINARIES

Definition(2.1): Abinary operation : $[0,1]x[0,1] \rightarrow [0,1]$ is a continuous t-norm if $\{[0,1],*\}$ is an abelian topological monoid with unit 1 such that $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, $a,b,c,d \in [0,1]$

Definition (2.2): A **fuzzy 2-metric space** is a space X in which for each triplet of points x,y,z there exits a real function M(x,y,z,t) such that :

(i)to each pair of distinct points x,y,z in X there exists a point t in[0,1] then $M(x,y,z,t)\neq 0$;

(ii)$M(x,y,z,t)=1$,when x,y,z are equal;

(iii)$M(x,y,z,t)= M(y,x,z,t)$;

(iv)$M(x,y,z,t_1+t_2+t_3)\geq M(x,y,z,t_1) * M(x,y,z,t_2) * M(x,y,z,t_3)$,

for all x,y,z in X and t_1, t_2, t_3 in [0,1].It is easily seen that M is non -negative.

Definition(2.3):A sequence$\{x_n\}$in a fuzzy 2-metric space(X,M,$*$)is said to be **convergent** with lim x in X if

$\lim_{n\to\infty} M(x_n,x,z,t)=1$,for all x,y,z in X and t>0.

Definition(2.4):A sequence $\{x_n\}$ in a fuzzy 2-metric space (X,M,$*$) is said to be a **Cauchy sequence** if

$\lim_{m,n\to\infty} M(x_m,x_n,z,t)=1$,for all x,y,z inX and t>0.

Definition(2.5):A fuzzy 2-metric space (X,M,$*$)is said to be **complete** fuzzy 2-**metric space** if every Cauchy sequence in X is convergent.

DEFINITION (2.6): Two self-mappings A and S of a fuzzy 2-metric space (X,M,$*$) are said to be compatible if,

$\lim_{n\to\infty} M(ASx_n, SAx_n,z, t) = 1$ whenever $\{x_n\}$ is a sequence such that,

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION (2.7): Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be weakly compatible if, they commute at coincidence points.That is, Ax= Sx implies that ASx=SAx for all x in X.

DEFINITION (2.8): Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be subcompatible if, $\lim_{n\to\infty} M(ASx_n, SAx_n, z, t) = 1$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION (2.9): Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be reciprocally continuous if,

$$\lim_{n\to\infty} M(ASx_n, Ax_n, z, t) = 1$$

$$\text{and } \lim_{n\to\infty} M(SAx_n, Sx_n, z, t) = 1$$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

It is noted that if A and S are both continuous, they are obviously reciprocally continuous but the converse need not be true.

DEFINITION (2.10): Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be compability of type(A) if,

$$\lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = 1$$

$$\text{and } \lim_{n\to\infty} M(SAx_n, AAx_n, z, t) = 1$$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION (2.11): Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be compability of type(B) if,

$$\lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = \frac{1}{2}[M(ASx_n, Ap, z, t) + M(Ap, AAx_n, z, t)]$$

$$\text{and } \lim_{n\to\infty} M(SAx_n, AAx_n, z, t) = \frac{1}{2}[M(SAx_n, Sp, z, t) + M(Sp, SSx_n, z, t)]$$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION (2.12): Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be compability of type(C) if,

$$\lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = \frac{1}{3}[M(ASx_n, Ap, z, t) + M(Ap, SSx_n, z, t) + M(Ap, AAx_n, z, t)]$$

$$\text{and } \lim_{n\to\infty} M(SAx_n, AAx_n, z, t) = \frac{1}{3}[M(SAx_n, Sp, z, t) + M(Sp, AAx_n, z, t) + M(Sp, SSx_n, z, t)]$$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION (2.13): Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be compability of type(D) if,

$$\lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = \frac{1}{4}[M(ASx_n, Ap, z, t) + M(Ap, AAx_n, z, t) + M(Ap, SSx_n, z, t) + M(SSx_n, AAx_n, z, t)]$$

$$\text{and } \lim_{n\to\infty} M(SAx_n, AAx_n, z, t) = \frac{1}{4}[M(SAx_n, Sp, z, t) + M(Sp, SSx_n, z, t) + M(Sp, AAx_n, z, t) + M(AAx_n, SSx_n, z, t)]$$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

3. MATERIAL AND METHOD

Theorem(3.1):Let $(X,M,*)$ a fuzzy 2-metric space and let T,A,B be continuous self mappings of X , then $\{Tx_n\}$,$\{Ax_n\}$and$\{Bx_n\}$ converges to p. If T,A and B satisfying the following condition

$$M(ATx, BTy, z, t) \le aM(x, y, z, t)$$
$$+b[M(x.ATx, z, t) + M(y, BTy, z, t)]$$
$$+c[M(x, BTy, z, t) + M(y, ATx, z, t)]$$

s.t. x,y,z in X and $t \in [0,1]$ when $a+2b+2c<1$, then T,A and B has a common fixed point p.

4. RESULT AND DISCUSSION

Proof : Consider $x=x_n$ and $y=x_{n+1}$

Also $ATx_n=Tx_{n+1}= x_{n+1}$

and $BTx_{n+1}=Tx_{n+2} =x_{n+2}$

Then above equation reduces to

$M(x_{n+1},x_{n+2},z,t) \le aM(x_n,x_{n+1},z,t)$

$+b[M(x_n,x_{n+1},z,t)+M(x_{n+1},x_{n+2},z,t)]$

$+c[M(x_n,x_{n+2},z,t)+M(x_{n+1},x_{n+1},z,t)]$

$\Rightarrow M(x_{n+1},x_{n+2},z,t) \le aM(x_n,x_{n+1},z,t)$

$+b[M(x_n,x_{n+1},z,t)+M(x_{n+1},x_{n+2},z,t)]$

$+c[M(x_n,x_{n+1},z,t)+M(x_{n+1},x_{n+2},z,t)$

$+M(x_{n+1},x_{n+1},z,t)]$

$M(x_{n+1},x_{n+2},z,t)$

$$\le \frac{a+b+c}{1-b-c} M(x_n,x_{n+1},z,t)$$

$\Rightarrow M(x_{n+1},x_{n+2},z,t) \le k\ M(x_n,x_{n+1},z,t)$

if $k=\frac{a+b+c}{1-b-c}$

Then

$M(x_{n+1},x_{n+2},z,t) \le k^n M(x_0,x_1,z,t) \to 1$

as $a+2b+2c<1$.

5. CONCLUSION

It is easily prove that

$\{Tx_n\}_{n=1}^{\infty}\ \{Ax_n\}_{n=1}^{\infty}\ \{Bx_n\}_{n=1}^{\infty}$ were a Cauchy sequence, since

$M(Tx,Ty,z,t) \le aM(x,y,z,t)$

$+b[M(x.Tx,z,t)+M(y,Ty,z,t)]$

$+c[M(x,Ty,z,t)+M(y,Tx,z,t)]$

$Tx_n= p$,$x=x_n$,$y=x_{n+1}$ and $a+2b+2c<1$.

Then p be fixed point of T. Similarly p be fixed point of A and B, So p is common fixed point of T,A and B. Also we can prove that above equation satisfy the condition of type(A),type(B),type(C),type(P) and type(D) by some rearrangement.

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