International Journal of Innovative Studies in Sciences and Engineering Technology (IJISSET)

ISSN 2455-4863 (Online) www.ijisset.org *Volume: 3 Issue: 3 | March 2017*

Coincidence Fixed Point in Hilbert Spaces

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Abstract: *In the present paper, we prove coincidence fixed point theorem for two pairs of contraction mappings in Hilbert spaces without any continuity requirement.*

Keywords and Phrases: *Hilbert space, concidence point, common fixed point Contraction mappings, commuting mapping, Cauchy sequence.*

AMS Subject classification: *Primary 47H10, Secondary 54H25*

1. INTRODUCTION

In recent years some fixed points of various type of compability mapping in Hilbert space and Banach spaces were obtained by Browder [1], Browder and Petryshyn [2], Hicks and Huffman [3], Jungck[4].

2. PRELIMINARIES

2.1 **NORM**: A norm on X is a real-valued function $||.||: X \to R$ defined on X such that for any $x, y \in X$ and for all $\lambda \in K$

(a) $||x|| = 0$ if and only if $x = 0$

(b) $||x+y|| \leq ||x|| + ||y||$

(c) $||\lambda x|| = |\lambda|\ ||x||$

2.2 **NORMED LINEAR SPACE**: It is a pair (X, ||.||) consisting of a linear space X and a norm ||.||. We shall abbreviate normed linear space as nls.

2.3 CAUCHY SEQUENCE: A Sequence $\{x_n\}$ in a normed linear space X is a Cauchy sequence if for any given $\varepsilon > 0$, there exist $n_0 \in N$ such that $||x_m - x_n|| < \varepsilon$ for $m, n \geq n_0$

2.4 **CONVERGENCE CONDITION IN NLS**: A sequence $\{x_n\}$ in a nls X is said to be Convergent to $x \in X$ if for any given $\varepsilon > 0, \exists\ n_0 \in N$ such that $||x_n - x|| < \varepsilon$ for $n \geq n_0$

2.5 **COMPLETENESS**: A nls X is said to be complete if for every Cauchy Sequence in X converges to an element of X.

2.6 **BANACH SPACE**: A Banach Space (X, ||.||) is a complete nls.

2.7 **INNER PRODUCT SPACE**: Let X be a linear space over the scalar field C of complex numbers. An inner product on X is a function $(.\ ,\ .) : XxX \to C$ which satisfies the following conditions

(a) $(x, y) = \overline{(y, x)}$ for $x, y \in X$

(b) $(\lambda x + \mu y, z) = \lambda\ (x, z) + \mu\ (y, z)$

for $\lambda, \mu \in C, x, y, z \in X$

(c) $(x, x) \geq 0$; x x) = 0 iff $x = 0$

2.8 **LAW OF PARALLELOGRAM:** If x and y are any two elements of an inner product space X then $||x + y||^2 + ||x-y||^2 = 2||x||^2 + 2||y||^2$

2.9 **HILBERT SPACE** : An infinite dimensional inner product space which is complete for the norm induced by the inner product is called Hilbert Space.

3. MATERIAL AND METHOD

Theorem: Let C be a closed subset of a Hilbert space H. $A,B,S,T:C \to C$ are contraction and continuous map of C , then A,B,S,T converges to u. If

$$\|Ax - By\|^2 \leq \max\{\|Sx - Ty\|^2, \|Sx - Ax\|^2,$$
$$\|By - Ty\|^2, \|Ax - Ty\|^2\ \|Sx - By\|^2\} \text{ (i)}$$

Also (i) (A,S) has coincidence point

(ii) (B,T) has coincidence point

(A,S),(B,T)are coincidently commuting then A,B,S and T have a unique common fixed point

4. RESULT AND DISCUSSION

Proof of theorem: Consider x_0 be an arbitrary point in X,since $A(X) \subset T(X)$,we can find a point x_1 in X s.t.

$Ax_0 = Tx_1$

Also B(X)$\subset$ S(X), we can choose a point x_2 with $Bx_1=Sx_2$

Then construct a sequence

$u_{2n}=Ax_{2n}=Tx_{2n+1}$

$u_{2n+1}=Bx_{2n+1}=Sx_{2n+2}$

if we substituting $x=x_{2n}$ and $y=x_{2n+1}$ then by (i)

$$\|Ax_{2n}-Bx_{2n+1}\|^2 \le \max\{\|Sx_{2n}-Tx_{2n+1}\|^2, \|Sx_{2n}-Ax_{2n}\|^2, \|Bx_{2n+1}-Tx_{2n+1}\|^2, \|Ax_{2n}-Tx_{2n+1}\|^2, \|Sx_{2n}-Bx_{2n+1}\|^2\}$$

$$\Rightarrow \|u_{2n}-u_{2n+1}\|^2 \le \max\{\|u_{2n-1}-u_{2n}\|^2, \|u_{2n-1}-u_{2n}\|^2, \|u_{2n+1}-u_{2n}\|^2, \|u_{2n}-u_{2n}\|^2, \|u_{2n-1}-u_{2n+1}\|^2\}$$

$\le \max\{\|u_{2n-1}-u_{2n}\|^2, \|u_{2n-1}-u_{2n}\|^2, \|u_{2n+1}-u_{2n}\|^2, \|u_{2n}-u_{2n}\|^2, 2\|u_{2n-1}-u_{2n1}\|^2 \ +2\|u_{2n}-u_{2n+1}\|^2\}$ by parallelogram law

$\le \|u_{2n+1}-u_{2n}\|^2$ Contradiction

$\Rightarrow$ u_{2n} is Cauchy sequence ,then claim that u_{2n} is convergent

$$\|Ap-Bx_{2n+1}\|^2 \le \max\{\|Sp-Tx_{2n+1}\|^2, \|Sp-Ap\|^2, \|Bx_{2n+1}-Tx_{2n+1}\|^2, \|Ap-Tx_{2n+1}\|^2, \|Sp-Bx_{2n+1}\|^2\}$$

Then

$$\|Ap-u\|^2 \le \max\{\|Ap-u\|^2, \|u-Ap\|^2, \|u-u\|^2, \|Ap-u\|^2, \|u-u\|^2\}$$

$\Rightarrow \|Ap-u\|^2 \le \|Ap-u\|^2$ Again contradiction

$\Rightarrow$ Ap=u=Sp ,but A(X)$\subset$ T(X)

So (i) u=Ap=Sp=Bq=Tq

(ii) Au=A(Sp)=S(Ap)=Su

(iii) Bu=B(Tq)=T(Bq)=Tu

Then

$$\|Au-Bx_{2n+1}\|^2 \le \max\{\|Su-Tx_{2n+1}\|^2, \|Su-Au\|^2, \|Bx_{2n+1}-Tx_{2n+1}\|^2, \|Au-Tx_{2n+1}\|^2, \|Su-Bx_{2n+1}\|^2\}$$

$$\Rightarrow \|Au-u\|^2 \le \max\{\|Au-u\|^2, \|u-Au\|^2, \|u-u\|^2, \|Au-u\|^2, \|u-u\|^2\}$$

$\Rightarrow \|Au-u\|^2 \le \|Au-u\|^2$ Contraction

5. CONCLUSION

From above Au=u=Su

Similarly Bu=u=Tu

Then u is unique coincidence fixed point. Suppose w is also a another fixed point of A,B,S and T then

$$\|u-w\|^2 = \|Au-Bw\|^2 \le \max\{\|Su-Bw\|^2, \|Su-Au\|^2, \|Tw-Bw\|^2, \|Au-Tw\|^2, \|Su-Bw\|^2\}$$

$\Rightarrow \|u-w\|^2 \le \|u-w\|^2$ Contradiction

ACKNOWLEDGEMENTS

The authors are thankful to the reviewers for their valuable suggestions to enhance the quality of our article and Journal also.

REFERENCES

[1] Browder, F.E.: Fixed point theorems for nonlinear semi contractive mappings in Banach space, Arch, Rat, Mech, Anal, 21,259-269,(1965- 66) .

[2] Browder, F.E. and Petryshyn W.V.: Contraction of fixed points of nonlinear mappings in Hilbert space ,J.Math. Anl. Appl.20,197-228, (1967).

[3] Hichs, T.L.and Huffman, Ed.W.: Fixed point theorems of generalized Hilbert space, J.Math Anal,Appl ,64 (1978).

[4] Jungck G.: Compatible mappings and common fixed points, Internet J. Math. and Math.Sci.9 (4) (1986), 771-779.

[5] Sao,G.S.: Common fixed point theorem for compability on Hilbert space,Applied Sci.Periodical,vol. 9(1),Feb.2007,p.27-29

[6] Sharma, Aradhana and Sao, G.S.: Meir Keeler type contractive conditions on Hilbert space,Acta Ciencia Indica vol.34(4)2008,p.1737-1738

[7] Sao,G.S. and Gupta S.N.:Common fixed point theorem in Hilbert space for rational expression, Impact Jour.of Sci.and Tech.vol. 4, 2010,P.B. No. 1889 Lautoka Fiji Island,p.39-41

[8] Sao, G.S. and Sharma Aradhana : Generalisation of Common fixed point Theorems of Naimpally and Singh in Hilbert Space, Acta Sciencia India 2008 34(4) p. 1733-34.

[9] Sharma, Aradhana and Sao, G.S. : Common Fixed Point in Banach Space International Journal of Modern Science and Engineering Technology Vol-2 Issue-8 2015 pp. 54-59.

[10] Sinha, Seema, Verma, Premlata and Sao, G.S.: Common fixed point in Hilbert space Global Jour. of engineering science and researches vol.2,issue 10(2015),pp. 86-88.

[11] Sao, G.S.,Sao, A.K., Sinha Seema and Shrivastava U.K.: compability and contraction mappings in Hilbert space International Jour.of innovative studies in Sci. Engineering and tech. ISSN No. 2455-4863,vol.3,issue 01,Jan 2017,p.4-6

[12] Yadav, Hema, Sayyed, S.A. and Badshah , V.H:, A note on common fixed point theorem in Hilbert space, *Material Science Research India*, vol.7 (2)(2010), 515-518.

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