

# A Family of Compensated Amplifiers for High Frequency Applications

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**Abstract:** A family of compensated difference amplifiers for high frequency of applications is presented. These amplifiers are classified into type I and Type II depending upon the location of the compensating network in the feedback or feed forward path. From these amplifiers compensated inverting and non-inverting amplifiers are derived. From type II amplifiers, amplifiers with zero-inverse-gain sensitivity are derived. Finally a method of realizing active networks with zero-transfer function sensitivity is suggested.

**Keywords:** Zero-gain sensitivity amplifiers, compensated finite-gain amplifiers, high frequency amplifiers

## 1. INTRODUCTION

Finite gain amplifiers find wide applications, such as in active RC filters, tuned amplifiers, oscillators, instrumentation, etc [1]-[5]. When these amplifiers are designed with operational amplifiers (OAs) assuming them ideal, their finite-gain bandwidths (GBs) adversely affect the high-frequency responses of these amplifiers and the circuits in which they are embedded [6]-[8]. For high-frequency applications of conventional amplifiers, passive and active compensation schemes have been reported [7]-[14]. Use of passive compensation is impractical due to frequent adjustment of the compensating elements in accordance with the variations in GBs of OAs, power supply voltages and temperature [15]. Actively-compensated finite-gain amplifiers with effects of finite GBs of OAs up to first and second order have appeared [9][12][13][14]. For convenience, E(n) will be used to mean 'effects of finite GB of OA up to nth order'. In the present paper, we propose a family of compensated amplifiers (CAs) some of which have zero-gain sensitivity and eliminate E(n).

## 2. TYPE I CAS

Consider the difference amplifier circuit shown in Fig.

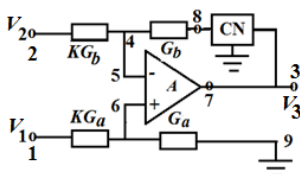


Fig 1: Difference amplifier circuit

1 where CN block represents a 3-terminal CN with voltage transfer function (VTF)  $T_{CN}$ . Assuming zero output impedance ( $Z_o$ ) for the CN, VTF can be expressed as

$$T = \frac{V_3}{V_1 - V_2} = \frac{K}{T_{CN} + (1 + K)/A} \quad (1)$$

Assuming single pole roll-off model for the OA, its gain

$$A = \omega_t / s = \frac{1}{s\tau}$$

where  $\omega_t = \frac{1}{\tau}$  represents the unity-gain band width of the OA. Substituting for A in eqn (1), one gets

$$T = \frac{K}{T_{CN} + (1 + K)s\tau} = \frac{K}{T_{CN} + \alpha s} \quad (2)$$

where

$$\alpha = (1 + K)\tau \quad (3)$$

For the circuit of Fig. 1 to act as a difference amplifier of gain K independent of frequency, eqn (2) dictates that

$$T_{CN} = (1 - \alpha s) \quad (4)$$

Thus the performance of the amplifier depends upon how accurately one realizes  $T_{CN}$  given by eqn (4).

To realize  $T_{CN}$  given by eqn (4) with the available physical elements, we first approximate  $T_{CN}$  suitably.

Eqn (4) can be rewritten, when  $\alpha\omega \ll 1$ , as

$$T_{CN} = \frac{1}{(1 - \alpha s)^{-1}} \cong \frac{1}{\sum_{i=0}^{\infty} (\alpha s)^i}$$

Again for practical realization of  $T_{CN}$ , we take first (n+1) terms of the infinite series. Thus

$$T_{CN} = \frac{1}{\sum_{i=0}^n (\alpha s)^i} \quad (5)$$

Substituting for  $T_{CN}$  from eqn (5) into eqn (2), one gets

$$T = \frac{K \sum_{i=0}^n (\alpha s)^i}{\sum_{i=0}^{n+1} (\alpha s)^i} \quad (6)$$

Thus  $T$  equals  $K$  if  $(\alpha s)^{n+1} = 0$ . Hence if the highest frequency of operation  $\omega_h$  is such that

$$[(1 + K)\omega_h / \omega_t]^{n+1} = P \cong 0. \quad (7)$$

$T$  given by eqn (6) would represent the ideal difference amplifier of gain  $K$ . Therefore compensation conditions (CCs) can be stated as

CC(i) Coefficients of all the powers of  $s$  in the numerator and denominator of  $T$  must be equal up to the  $n$ th order term of  $s$ .

CC(ii) Eqn (8) must hold. i.e.,  $\omega_h \ll \omega_t$ .

In view of eqn (5), eqn (7) is satisfied for large values of  $n$ . In practice  $n = 2$  is sufficient. As an illustration, taking  $K = 10$  and  $\omega = 0.01\omega_b$ ,  $P$  equal 0.001331 for  $n = 2$  and 0.000146 for  $n = 3$ . The latter figure is negligible compared to the former one.

Total change  $\Delta T$  in  $T$  due to changes in  $\alpha_i s$  is given by

$$\Delta T = \frac{\partial T}{\partial \alpha} \Delta \alpha + \frac{\partial T}{\partial \alpha_1} \Delta \alpha_1 + \frac{\partial T}{\partial \alpha_2} \Delta \alpha_2 + \dots + \frac{\partial T}{\partial \alpha_n} \Delta \alpha_n. \quad (8)$$

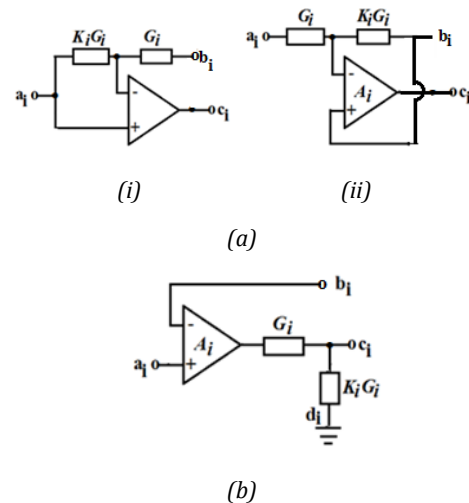
It is interesting to note that  $T$  given by eqn (6) gives  $\Delta T = 0$  due to departures in  $\alpha_i s$  around their ideal values of zero. Thus, Fig. 1 represents a zero-gain-sensitivity difference amplifier (ZGSDA) when CN of Fig. 3 is embedded into it and also has  $E(n)$  eliminated.

### 2.1 Active realization of $T_{CN}$

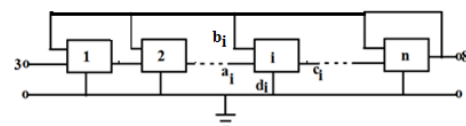
If all the OAs in Fig. 1, including those used in the CN, were ideal, eqn (1) would demand that  $T_{CN} = 1$  to yield  $T = K$ . Hence for an  $n$ th order compensator the obvious choice is  $n$  conventional unity-gain amplifiers connected in cascade. The voltage transfer function of such a CN is

$$T_{CN} = \frac{V_8}{V_3} = \frac{1}{\prod_{i=1}^n (1 + s\tau_i)}, \quad s\tau_i = \frac{1}{\omega_{ii}}$$

It can easily be seen that CC(i) demands the adjustments of  $\tau_i s$  of the various OAs. For example when  $n = 1$ ,  $\tau_1 = \tau$  which is difficult to satisfy in practice when the OAs are not matched. This difficulty may be overcome if the  $\tau_i s$  are weighted by suitable factors. One possible way is to replace the unity-gain amplifiers with any one type of sub-networks (SNs) (of unity gain when the OAs are ideal) shown in Fig. 2: 2 in Fig. 2(a) and 1 in Fig. 2(b). They are designated as Type A1 and A2, and Type B SNs respectively. Type A SNs are floating and have zero output impedance; while Type B SN is grounded and has finite output impedance. The VTF of any of these CNs is



**Fig 2:** Subnetworks (a) (i) Type A1 (ii) Type A2, (b) Type B



**Fig 3:** Compensating Network

$$T_{CN} = \frac{V_8}{V_3} = \frac{1}{\prod_{i=1}^n (1 + \alpha_i s)}, \quad \alpha_i = (1 + k_i)\tau$$

which can be expressed as

$$T_{CN} = \frac{1}{1 + a_1 s + \dots + a_1 a_2 \dots a_n s^n}, \quad (9)$$

Now CC(i) will demand  $a_i = \alpha^i$  ( $i = 1$  to  $n$ ). These  $n$  relations can simultaneously be solved for the  $K_i$  which is not only difficult but it may also be possible

that they may not be real. Similar situation may arise if some  $b_j$ s are connected to  $c_k$ s ( $k > j$ ). However, guaranteed non-negative real  $K$ s, as will be shown shortly, if all the  $b_j$ s are connected to  $c_n$  as shown in Fig. 3.

The transfer function of the CN of Fig. 3, when any of the SNs of Fig. 2 are connected in any combination (Type B SNs should have a suitable buffer amplifier at its output), is

$$T_{CN}(s) = \frac{1}{1 + \alpha_1 s + \alpha_1 \alpha_2 s^2 + \dots + \alpha_1 \alpha_2 \dots \alpha_n s^n}.$$

Comparing with  $T_{CN}$  given by eqn (5) we get

$$\alpha_i = \alpha \text{ or } K_i = (1 + K)\tau / \tau_i - 1, \quad i = 1 \text{ to } n. \quad (10)$$

$K$ is in eqn (10) may be suitably chosen to accommodate different  $\tau_i$  of non-matched OAs. If OAs are matched, eqn (9) reduces to  $K_i = K$ . As the different OAs will be fabricated on the same chip, at the same time, using the same processing steps, it is reasonable to assume that the  $\tau_i$ s are equal.

Since Type A1 and A2 SNs have  $Z_o = 0$ , they can be used without any buffer network in Fig. 1 in variety of combinations, giving several possible ZGSDAs each consisting of  $(n + 1)$  OAs and  $(2n + 4)$  resistors. These CAs are referred to Type I-A. Note that Soliman and Ismail Circuits [12][13] are the special cases of these generalized CAs.

Since Type B SNs has  $Z_o$  not equal to 0, a compensated buffer amplifier at the output of CN of Fig. 2 (b) is required when it is embedded in Fig. 1. Thus the resulting difference amplifier would require more than  $(n + 1)$  OAs.

### 3. TYPE II CAS

If the output in Fig. 1 is taken at node 8, new VTF is

$$T = \frac{K}{[(1 + \alpha s / T_{CN})]}. \quad (11)$$

Ideally,  $T_{CN}$  should be such that  $\alpha s / T_{CN} = 0$  to yield  $T = K$ . However, for practical realization of  $T$ , we approximate

$$T_{CN} = \frac{1}{\mu s^n} \quad (12)$$

where  $\mu$  is a very very small real number, so that  $\frac{\alpha s}{T_{CN}} \cong 0$  within the frequency range of operation.

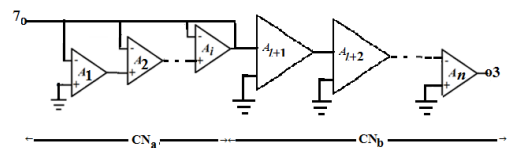
Consider the CN shown in Fig. 4. The transfer functions of CNa and CNb networks are respectively

$$T_{CNa}(s) = \frac{1 + \tau_1 s + \dots + \tau_1 \tau_2 \dots \tau_{j-1} s^{j-1}}{\tau_1 \tau_2 \dots \tau_n s^j}. \quad (13)$$

$$T_{CNb}(s) = \frac{1}{\tau_{j+1} \tau_{j+2} \dots \tau_n s^n}. \quad (14)$$

Overall transfer function

$$T_{CN}(s) = \frac{1 + \tau_1 s + \dots + \tau_1 \tau_2 \dots \tau_{j-1} s^{j-1}}{\tau_1 \tau_2 \dots \tau_n s^n}. \quad (15)$$



**Fig 4: Compensating network**

From eqns (11) and (15), we get

$$T(s) = \frac{K(1 + \tau_1 s + \dots + \tau_1 \tau_2 \dots \tau_{j-1} s^{j-1})}{1 + \tau_1 s + \dots + \tau_1 \tau_2 \dots \tau_n s^n + \alpha \tau_1 \tau_2 \dots \tau_n s^{n+1}} \cong K. \quad (16)$$

From eqn (16) and Fig. 4, we note the following.

- i) The numerator and denominator coefficients of  $s^\mu$  ( $\mu = 0$  to  $j$ ) in  $T$  are equal and those of  $s^\mu$  ( $\mu = j$  to  $n$ ) are zero. The coefficient of  $s^{j+1}$  can also be made zero by pushing OA  $A_j$  from CNa into CNb and connecting it in the same manner as  $A_{j+1}$ .
- ii) All the coefficients up to  $\mu \leq n - j$  can be made non-zero having  $j$  OAs in the CNa and  $(n - j)$  OAs in CNb.
- iii) In view of (i) and (ii) above there are many possible Type II CAs (restricting the case that has only one OA in CNa in the entire cascade chain in the CN), each requiring  $N+1$  OAs and no more additional resistors than are needed in the conventional difference amplifier with an ideal OA. Component-wise type II CAs are better than those of type I. Moreover trimming of resistors for accommodating the  $\tau_i$ s of non-matched OAs is eliminated.
- iv) CNa must have at least two OAs.
- v) CNa can be inserted after any of the OAs in CNb.

vi) The last term in (16) is negative, can be made positive by interchanging the input terminals of any one OA in CN.

If CNb alone is used, then from eqns (11) and (14) we get

$$T = \frac{K}{1 + \tau_1\tau_2\dots\tau_n\alpha s^{n+1}} \quad (17)$$

Since  $\tau_1\tau_2\dots\tau_n\alpha \ll 1$ ,  $T = K$ . This amplifier is referred to Type IIA CA.

If CNa alone is used, then from eqns (11) and (13) we get

$$T(s) = \frac{K(1 + \tau_1s + \dots + \tau_1\tau_2\dots\tau_{j-1}s^j)}{1 + \tau_1s + \dots + \tau_1\tau_2\dots\tau_n s^n + \alpha\tau_1\tau_2\dots\tau_n s^{n+1}} \quad (18)$$

Since  $\alpha\tau_1\tau_2\dots\tau_n s^{n+1} \cong 0$ ,  $T = K$ . This amplifier is referred to Type IIB CA. Note that it has zero-gain sensitivity and eliminates E(n)..

#### 4. COMPENSATED INVERTING AMPLIFIERS (CIAS)

CIAs can be obtained from Types I and II CAs derived above by shorting terminal 1 and ground. In all these cases redundant L-network,  $KG_a-G_a$ , can be eliminated by grounding terminal 6.

All possible CIAs for  $n = 2$  are shown in Fig. 5. Note the following.

i) Fig. 5(c)(ii) and 5(d)(ii) are obtained from 5(c)(i) and 5(d)(i) respectively by interchange-

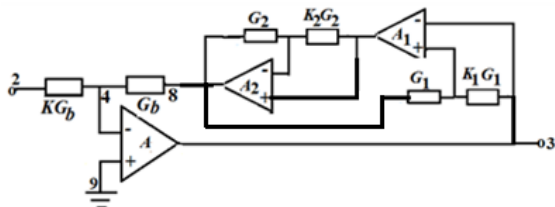


Fig. 5(a)(i)

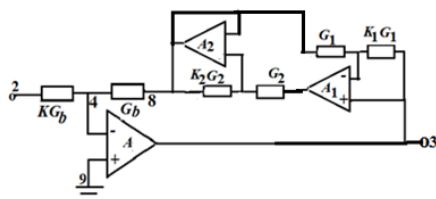


Fig. 5(a)(ii)

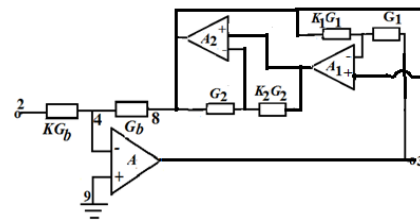


Fig. 5(a)(iii)

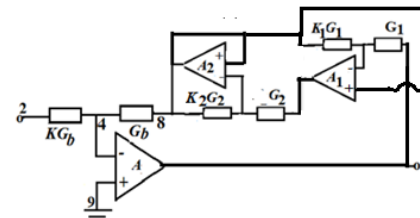


Fig. 5(a)(iv)

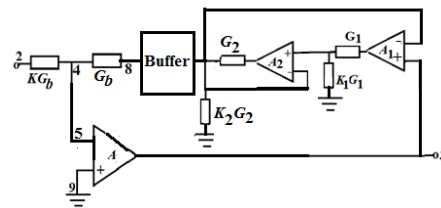


Fig. 5(b)

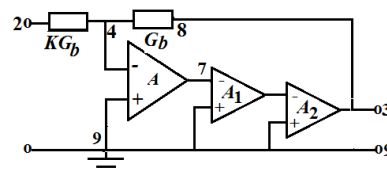


Fig. 5(c)(i)

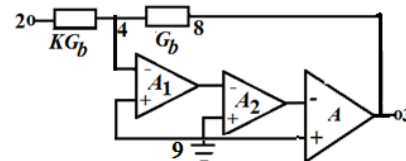


Fig. 5(c)(ii)

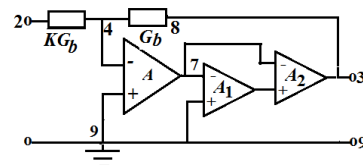


Fig. 5(d)(i)

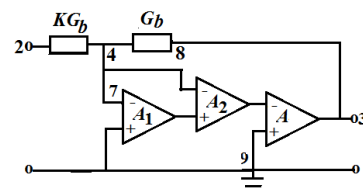


Fig. 5(d)(ii)

**Fig 5:** CIAs (a) Type IA, (b) Type IB, (c) Type IIA (d) Type IIB

ing the positions of the main and compensating amplifiers. Since these amplifiers are connected in cascade, do not change the overall gain.

- ii) Type II CIAs require less number of resistors than those of Type I CIAs.
- iii) Except Type IIA CIAs, all have zero-gain sensitivity to the changes in  $\alpha_i$ s around their normal values of 0.

### 5. COMPENSATED NON-INVERTING AMPLIFIERS (CNIAS)

CNIAs can be obtained from CIAs by interchanging the input terminal 2 and non-inverting terminal of amplifier A. This will modify the gain to  $(1 + K)$ . These amplifiers require less number of resistors than that proposed in [16]. Types IB amplifier with  $n = 1$  reduce to those proposed in [9].

### 6. REALIZATION OF ACTIVE NETWORKS WITH ZERO-TRANSFER FUNCTION SENSITIVITY

Type II CAs can be considered as the special cases of Fig. 6 where  $A_c$  is the VTF of the network consisting of CN and OA and is, therefore, function of  $\tau_i$ s. Since these amplifiers have zero gain sensitivity,  $\frac{\partial}{\partial \tau_i} \left[ \frac{1}{A_c} \right]$  must be zero [14]. This  $A_c$  will be referred to 'zero-inverse-gain sensitivity amplifiers' (ZIGSAs).

ZIGSAs can be considered almost ideal OAs [14]. Thus any active RC network with zero-transfer function sensitivity can be realized as follows. First realize the specified VTF by employing ideal OAs by any known method and then replace each OA by suitable (single ended or differential input type) ZIGSA. Specific active filters and integrators [14][18][19][20] with excellent performance are direct consequence of this systematic approach.

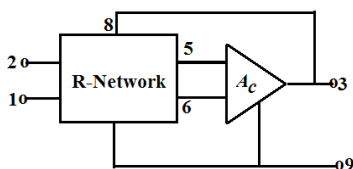


Fig 6: A general configuration

### 7. CONCLUSION

A family of compensated difference amplifiers suitable for high frequency applications has been proposed. They are classified as Type I and Type II respectively based on the location of the CN in the feedback or feed-forward path. Some of these CAs have  $E(n)$  eliminated. From all these compensated difference amplifiers,

inverting and non-inverting CAs have been derived. From Type II compensated difference amplifiers, ZIGSAs have been derived. Finally, a method of realizing active RC networks with zero-transfer function sensitivity employing non-ideal and non-matched OAs have been suggested.

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