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A Family of LC Oscillators

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Abstract: *A family of LC oscillators is derived. An oscillator basically consists of an amplifier of gain A and a passive network N in its feedback path. When A is negative real one additional buffer amplifier is required. It is shown that if OA-based amplifier is used for realizing A, the oscillators are such that their performance is not influenced by the input and output impedances of the OA. We first derive all simplest possible 6 oscillators when N is a π-network. Another set of 6 oscillators, when N is a T-network, is then obtained by L-C transformation. Within these 12 of oscillators, some can be derived from others by network transformations. Finally classification of the derived oscillators based on various factors is given.*

Keywords: *LC oscillators, L-C transformation, π-network, oscillators, T-network*

1. INTRODUCTION

There has been an interest in deriving oscillators from one topology to the other in the past [1][3]. Recently some work has appeared in the design of LC oscillators [4][5]. Dutta Roy [5] has given a general theory of LC oscillators from the configuration shown in Fig. 1 where amplifier of gain A is shown by its equivalent circuit and Z_i and Z_o as the *input and output resistances R_i and R_o of the amplifier,* and N as a network *with all impedances as pure reactances*. Thus the author deals with only a *specific type of impedances*.

In the present paper, we start with a most general 2-port passive network N and OA-based amplifier, and show that all elements in N must be reactive elements for oscillations. We get all the 12 simplest possible oscillators systematically and not by intuition. It is also shown that the set of 6 oscillators when N is a π-type network can be easily converted into another set of 6 oscillators when N is a T-type. There is a close

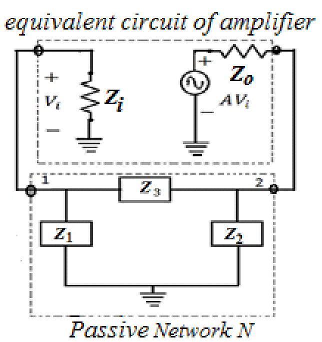

Fig 1: *Basic oscillator circuit with equivalent circuit of amplifier and passive network N as π network*

relationship among As and ω_os (oscillation frequencies) of the two sets. Further the design is such that one need not bother about the influence of input and output impedances of the amplifier at all. Finally all the oscillators are classified in different groups.

2. SYNTHESIS OF OSCILLATORS

Consider the configuration shown in Fig. 1. Since a 2-port passive network N can be represented by either **π**- or T-network, we have two possible types for N; one shown in Fig. 1 with **π**-network and the other in Fig. 5 with T-network.

A. N as a π network

In Fig. 1, Z_o cannot be zero otherwise Z_2 will appear across an ideal voltage source and will not be effective. Also it cannot be infinite for the circuit to work.

Analysis of the circuit leads to the following expression for the gain of the amplifier required for oscillation.

$$A=\left(1+\frac{Z_3}{Z_1}\right)+\frac{Z_o}{Z_i}\left[1+\frac{Z_3}{Z_2}\right]+\left(\frac{Z_1+Z_2+Z_3}{Z_1Z_2}\right)Z_o+\frac{Z_3}{Z_i} \quad (1)$$

This gain expression is very complicated. However, if $Z_i \to \infty$, (1) reduces to

$$A=\left(1+\frac{Z_3}{Z_1}\right)+\left(\frac{Z_1+Z_2+Z_3}{Z_1Z_2}\right)Z_o \quad (2)$$

Z_i is infinity for the OA-based non-inverting amplifier and not for the inverting one. However, in the latter case, one can use a buffer between the output of N and input of A to achieve infinite Z_i. In order to make the gain independent of Z_o, we choose

$$Z_1+Z_2+Z_3=0. \quad (3)$$

Now

$$A=\left(1+\frac{Z_3}{Z_1}\right). \quad (4)$$

For reduced complexity of the oscillator circuit, we assume that each impedance consists of single element. Equation (3) dictates that Z_1, Z_2 and Z_3 must be

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reactances such that at least one of them is of opposite nature than the other two. Thus there are three possible cases, namely Case A: Z_1 and Z_2, Case B: Z_2 and Z_3 and Case C: Z_3 and Z_1 are of similar nature either $j\omega L$ or $-j/\omega C$ type. Thus the passive network will consists of only reactances. Further, condition in (3) will be satisfied for one particular frequency ω_o called resonance frequency or frequency of oscillation.

From (3)

$$A = -\frac{Z_2}{Z_1} \tag{5}$$

Equations (3) and (5) are the basic relations for designing the LC oscillators. Though these relations do not depend upon Z_i, however, it has to be infinite as discussed above.

It is worth noting the difference between the approach we have followed and that in [5]. One can easily check that the gain expression (1) is the same as reported in [5] except for the Ys in place of Zs and R_o and R_i in place of Z_o and Z_i respectively. Since in [5] all the impedances are taken as reactive elements right in the beginning, the gain expression contains the real and imaginary parts. Then the real and imaginary parts are equated to get the condition of oscillation and frequency of oscillations.

In our approach, we assumed Z_i to be infinite (which can always be satisfied, using a buffer if required) which simplifies the gain expression. Then to eliminate the effect of R_o on the gain we choose the condition (3). Here we are not making $R_o = 0$ but eliminating its effect on gain. In [5] condition (3) decides the condition of oscillations, in our case it makes the gain independent of R_o.

Case A: Z_1 and Z_2 are of similar nature

Case A1: Both Z_1 and Z_2 capacitors

Let $Z_1 = -j/\omega C_1$ and $Z_2 = -j/\omega C_2$. Then from (3),

$$Z_3 = j\left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2}\right) = j\omega L_3 \text{ say} \tag{6}$$

This gives

$$\omega_O{}^2 = \frac{C_1 + C_2}{C_1 C_2 L_3} \tag{7}$$

From (5), we get

$$A = -\frac{C_1}{C_2}. \tag{8}$$

Case A2: Both Z_1 and Z_2 inductors

Let $Z_1 = j\omega L_1$ and $Z_2 = j\omega L_2$. Then from (3),

$$Z_3 = -j\omega(L_1 + L_2) = -j\frac{1}{\omega C_3}, \text{ say.} \tag{9}$$

This gives

$$\omega_O{}^2 = \frac{1}{(L_1 + L_2)C_3}. \tag{10}$$

From (5)

$$A = -\frac{L_2}{L_1} \tag{11}$$

From (8) and (11), the As are real, negative and independent of the frequency of oscillation. A being negative, additional buffer amplifier is to be used as suggested above. The

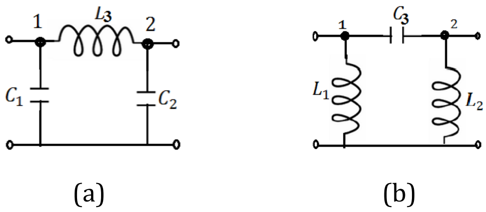


Fig 2: *Networks N (a) Case A1 (b) Case A2*

corresponding passive networks are shown in Fig. 2(a) and (b). Corresponding oscillators are popularly known as Colpitts and Hartley Oscillators respectively.

Case B: Z_2 and Z_3 of similar nature

Case B1: Both Z_2 and Z_3 capacitors

Let $Z_2 = -j/\omega C_2$ and $Z_3 = -j/\omega C_3$. Then from (3),

$$Z_1 = j\left(\frac{1}{\omega C_2} + \frac{1}{\omega C_3}\right) = j\omega L_1 \text{ say} \tag{12}$$

This gives

$$\omega_O{}^2 = \frac{C_2 + C_3}{C_2 C_3 L_1} \tag{13}$$

From (5)

$$A = \frac{1}{\omega^2 L_1 C_2}.$$

At $\omega = \omega_O$

$$A = -\frac{C_3}{C_2 + C_3}.. \tag{14}$$

Case B2: Both Z_2 and Z_3 inductors

Let $Z_2 = j\omega L_2$ and $Z_3 = j\omega L_3$. Then from (3),

$$Z_1 = -j\omega(L_2 + L_3) = -j/\omega C_1, \text{ say}. \quad (15)$$

This gives

$$\omega_O{}^2 = \frac{1}{C_1(L_2 + L_3)} \quad (16)$$

From (5)

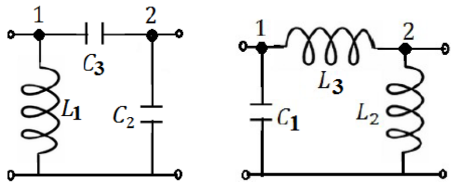
$$A = \omega^2 L_2 C_1.$$

At $\omega = \omega_O$

$$A = -\frac{L_2}{L_2 + L_3}. \quad (17)$$

$$\omega_O{}^2 = \frac{1}{C_1(L_2 + L_3)}$$

The corresponding passive networks are shown in Fig. 4. From (14) and (17), the *A*s are real, negative but less than 1, an attenuator, in place of an amplifier, preceded by a buffer will be required. As suggested above. The corresponding passive networks are shown in Fig. 3.



(a) *(b)*

Fig 3: *Networks N for (a) Case B1 and (b) Case B2*

Case C: Z_3 and Z_1 of similar nature

Case C1: Both Z_3 and Z_1 capacitors

Let $Z_3 = -j/\omega C_3$ and $Z_1 = -j/\omega C_1$. Then from (3),

$$Z_1 = j\left(\frac{1}{\omega C_2} + \frac{1}{\omega C_3}\right) = j\omega L_2 \text{ say} \quad (18)$$

This gives

$$\omega_O{}^2 = \frac{C_1 + C_3}{C_1 C_3 L_2} \quad (19)$$

From (5)

$$A = \frac{1}{\omega^2 C_1 L_2}.$$

At $\omega = \omega_O$

$$A = \frac{C_1 + C_3}{C_3}. \quad (20)$$

Case C2: Both Z_3 and Z_1 inductors

Let $Z_3 = \omega L_3$ and $Z_1 = j\omega L_1$. Then from (3),

$$Z_1 = -j(\omega L_1 + \omega L_3) = -j/\omega C_2 \text{ say} \quad (21)$$

This gives

$$\omega_O{}^2 = \frac{1}{C_2(L_1 + L_3)}. \quad (22)$$

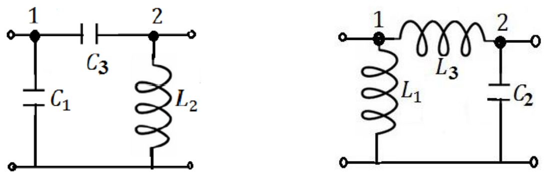
From (5)

$$A = \frac{1}{\omega^2 L_1 C_2}.$$

At $\omega = \omega_O$

$$A = \frac{L_2 + L_3}{L_1}. \quad (23)$$

The corresponding passive networks are shown in Fig. 4. From (20) and (23), note that the *A*s being real, positive greater than 1, a non-inverting amplifier will suffice.



(a) *(b)*

Fig 4: *Networks N for (a) Case C1 and (b) Case C2*

Effect of finite value of R_o

It is interesting to know that the finite value of Z_o does not have any effect on ω_o and *A*. This is shown below for Colpitts Oscillator of Fig. 2(a) for which

$$Z_1 = 1/sC_1,\ Z_2 = 1/sC_2, Z_3 = sL_3, Z_o = R_o.$$

Substituting these values in (2), we get

$$s^3 + \frac{1}{C_2 R_o}s^2 + \frac{C_1 + C_2}{C_1 C_2 L_3}s + \frac{1 - A}{C_1 C_2 L_3 R_o} = 0$$

$$s^2\left(s + \frac{1}{C_2 R_o}\right) + \frac{C_1 + C_2}{C_1 C_2 L_3}\left(s + \frac{1 - A}{(C_1 + C_2)R_o}\right) = 0$$

If we choose $\dfrac{1}{C_2} = \dfrac{1 - A}{(C_1 + C_2)} \Rightarrow A = -\dfrac{C_1}{C_2}$ then

$$\left(s+\frac{1}{C_2R_o}\right)\left(s^2+\frac{C_1+C_2}{C_1C_2L_3}\right)=0 \tag{24}$$

This can be satisfied by making the second factor = 0, i.e.,

$$s^2+\frac{C_1+C_2}{C_1C_2L_3}=0 \Rightarrow \omega_o{}^2=\frac{C_1+C_2}{C_1C_2L_3}$$

Thus the first factor does not affect $\omega_o{}^2$. Hence R_o has no effect on gain A and the frequency of oscillation $\omega_o{}^2$. similar results can be obtained for the oscillator circuits corresponding to remaining passive networks of Figs. 2(b), 3, and 4.

Transfer functions of passive networks

The transfer function of the passive network shown in Fig. 1 can be written as

$$\beta(s)=\frac{Z_1Z_2}{Z_1Z_2+Z_2Z_1+R_O(Z_1+Z_2+Z_3)} \tag{25}$$

For the passive networks shown in Figs. 2, 3 and 4 this becomes, respectively,

$$\beta_{A1}(s)=\frac{\left(\frac{1}{C_1C_2L_3R_o}\right)}{s^3+\frac{1}{C_2R_o}s^2+\frac{C_1+C_2}{C_1C_2L_3}s+\frac{1}{C_1C_2L_3R_o}} \tag{26}$$

$$\beta_{A2}(s)=\frac{s^3}{s^3+\frac{L_1+L_2}{L_1L_2}R_os^2+\frac{1}{L_1C_3}s+\frac{R_o}{L_1L_2C_3}} \tag{27}$$

$$\beta_{B1}(s)=\frac{\frac{1}{C_2R_o}s^2}{s^3+\frac{1}{C_2R_o}s^2+\frac{C_2+C_3}{L_1C_2C_3}s+\frac{1}{L_1C_2C_3R_o}} \tag{28}$$

$$\beta_{B2}(s)=\frac{\frac{1}{C_1L_3}s}{s^3+\left(\frac{L_2+L_3}{L_2L_3}R_o\right)s^2+\frac{1}{L_1C_3}s+\frac{R_o}{C_1L_2L_3}} \tag{29}$$

$$\beta_{C1}(s)=\frac{\frac{1}{C_1R_o}s}{s^2+\left(\frac{(C_1+C_3)L_2}{C_1C_3R_o}\right)s+\frac{C_1+C_3}{C_1C_3L_2}} \tag{30}$$

$$\beta_{C2}(s)=\frac{\left(\frac{L_1}{(L_1+L_3)C_2R_o}\right)s}{s^2+\left(\frac{1}{C_2R_o}\right)s+\frac{1}{(L_1+L_3)C_2}} \tag{31}$$

From the above relations it can be noted that the transfer functions of case A and case B types of passive networks are of 3rd order while those of case C type of passive networks are second order.

B. N as a T-network

If we try to find out T-equivalents of $\boldsymbol{\pi}$-networks shown in Figs. 2, 3, and 4, we will have three T-impedances but not necessarily they will contain single element. Therefore we cannot find simple network by $\boldsymbol{\pi}$-T (or delta-star) transformation.

Consider the equivalent circuit shown in Fig. 5 with N as a T-network. Analysis leads to

$$A=\left(1+\frac{Y_3}{Y_1}\right)+\frac{Z_o}{Z_i}\left[1+\frac{Y_3}{Y_2}\right]+\frac{1}{Z_i}\left(\frac{Y_1+Y_2+Y_3}{Y_1Y_2}\right)+Z_oY_3. \tag{32}$$

For an OA $Z_o \rightarrow 0$, so that

$$A=\left(1+\frac{Y_3}{Y_1}\right)+\frac{1}{Z_i}\left(\frac{Y_1+Y_2+Y_3}{Y_1Y_2}\right). \tag{33}$$

The gain can be made independent of Z_i by choosing

$$Y_1+Y_2+Y_3=0. \tag{34}$$

Now (22) simplifies to

$$A=1+\frac{Y_3}{Y_1}=-\frac{Y_2}{Y_1}. \tag{35}$$

Comparing (34) and (35) with (3) and (4), we find that Z_j are replaced by Y_j. For example, network shown in Fig. 2(a) will be reduced to that shown in Fig. 6(a) where $Y_1 = 1/Z_1 = -j/\omega L_1 = -j/\omega C_1$. This means L_1 in network should be C_1 in T-network and vice versa. Thus $L_j \Leftrightarrow C_j$. The T–equivalents of all the π-networks shown in Figs. 2, 3, and 4 obtained by L-C transformation are shown in Figs. 6, 7 and 8 respectively.

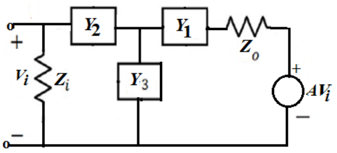
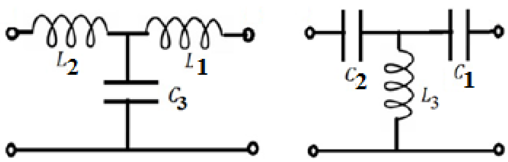
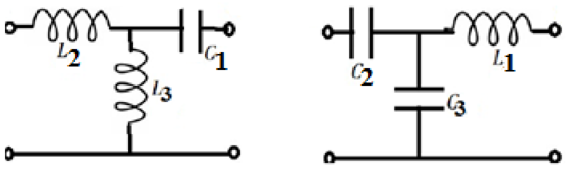


Fig 5: *Equivalent circuit of oscillator with N as a T network*



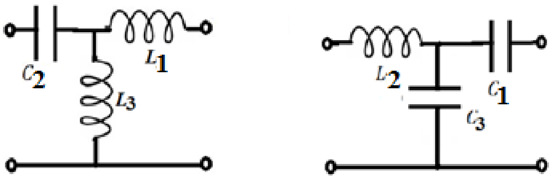
(a) (b)

Fig 6: *Networks N obtained from Fig. 2 (a) and (b)*



(a) (b)

Fig 7: *Networks N obtained from Fig. 3 (a) and (b)*



(a) (b)

Fig 8: *Networks N obtained from Fig. 4 (a) and (b)*

Their gains and frequencies of oscillation can be obtained from the corresponding relations by L-C transformation.

It is interesting to note that the pair of passive networks of Figs. 2, 3, 4, 6, 7 and 8 are dual of each other.

Further to note that the π-networks of Figs. 2(a) and 2(b) can be converted to π-networks of Figs. 3(a) and 3(b) respectively by interchanging input and ground terminals. This will change the voltage transfer of the passive network from β to (1-β) and therefore the corresponding gain of the amplifier required will be $1/(1-\beta)|_{\omega=\omega_O}$ [6]. Similarly, T-networks of Figs. 6(a) and (b) can be converted into T-networks of Figs. 7(a) and (b), respectively [6].

3. CLASSIFICATIONS OF LC OSCILLATORS

All the 12 oscillators are classified and are summarized in Table 1.

C. Some remarks

1. Since a pure inductor cannot be realized in practice, 2-inductor realizations should be avoided. Moreover, at high frequency, these inductors will have electro-magnetic coupling and therefore proper screening will be necessary.
2. 2-OA oscillators will continuously draw more power than single OA ones. Thus one should prefer 1-OA and 1-L circuits, i.e., with N given in Figs. 3(a), 4(a), 7(b), 8(b).
3. Further, corresponding to N of Figs. 3 and 7, the oscillators use a buffer amplifier of unity gain. Hence they will perform better for a large range of frequencies.
4. In case of N corresponding to Fig. 7(b), the effect of the series resistance of non-ideal inductor L_1 can be eliminated by absorbing it in R_o.

Table 1: *Classification of Oscillators*

Type	Based on	Circuits for N
I	Amplifier Gain A:	
	(1) negative gain	Figs. 2, 6
	(2) Positive gain < 1	Figs. 3, 7
	(3) Positive gain > 1	Figs. 4, 8
II	Number of OA	
	(1) One	Figs. 3, 4, 6, 7, 8
	(2) Two	Fig. 2
III	Frequency of oscillation $\omega_O{}^2$	
	(1) $\frac{C_a + C_b}{C_a C_b L_c}$	Figs. 2(a), 3(a), 4(a)
	(2) $\frac{1}{(L_a + L_b)C_c}$	Figs. 2(b), 3(b), 4(b)
	(3) $\frac{1}{(C_a + C_b)L_c}$	Figs. 6(b), 7(b), 8(b)
	(4) $\frac{L_a + L_b}{L_a L_b C_c}$	Fig. 6(a), 7(a), 8(a)
IV	Type of network N	
	(1) π	Figs. 2, 3, 4
	(2) T	Figs. 6, 7, 8
V	Number and types of elements in N	
	(1) 1 L and 2 C	Figs. 2(a), 3(a), 4(a), 6(b), 7(b), 8(b)
	(2) 2 L and 1 C	Figs. 2(b), 3(b), 4(b), 6(a), 7(a), 8(a)
VI	Order of N	
	(1) 3rd order	Figs. 2(a), 2(b), 3(a), 3(b), 6(a), 6(b), 7(a) 7(b)
	(2) 2nd order	Figs. 4(a), 4(b), 8(a), 8(b)

4. CONCLUSION

A systematic procedure for realizing a family of LC oscillators has been given. If OA-based amplifier is used in the oscillators then the performance of the oscillators are not influenced by the input and output impedances of the amplifier circuit. All simplest possible 6 oscillators are obtained through the procedure when N is a π-network and another set of 6 are obtained by L-C transformation on these π-networks. All the 12 oscillators have been classified into various groups based on various factors. Oscillator that uses N of Fig. 3(a) may be the best because it uses only 1 OA that too of unity gain, only 1 inductor, covers large range of frequencies.

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