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Contraction and Compability Mapping in Fuzzy 2-Metric Space

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Abstract: *The aim of present paper is to obtain a fixed point by contraction and compability in fuzzy 2-metric space .Our purpose is generalize the several known results.*

Keywords and Phrases: *fuzzy 2-metric spaces, contraction mapping, compability, fixed point.*

AMS subject classification (2000): *54H25, 47H10.*

1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [14] in 1965, who introduced the concept of fuzzy set. George and Veeramani [3] modified the notion of fuzzy metric spaces by introducing the concept of continuous t – norm.Now define fuzzy 2-metric space. The fuzzy metric space was introduced by Kramosil and Michalek [5],Khan M.S, Pathak H.K. & George R.[6] compatible mappings of Type (A–1) and Type A–2 in fuzzy metric spaces, Sao,G.S. and Shrivastava,U.K. [11] introduce the notion of compability of type(D)

2. PRELIMINARIES

The following definitions and results will be needed in the sequel.

DEFINITION 2.1([13]) Let X be any set. A fuzzy set A in X is a function with domain X and values in [0, 1].

Definition(2.2)[3]:Abinary operation : $[0,1]x[0,1] \to [0,1]$ is a continuous t-norm if {[0,1],*} is an abelian topological monoid with unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $a,b,c,d \in [0,1]$

Definition(2.3)[3]:A **fuzzy 2-metric space** is a space X in which for each triplet of points x,y,z there exists a real function M(x,y,z,t) such that :

(i) to each pair of distinct points x,y,z in X there exists a point t in[0,1] then $M(x,y,z,t)\neq 0$;

(ii) M(x,y,z,t) = 1, when x,y,z are equal;

(iii) M(x,y,z,t)= M(y,x,z,t) ;

(iv) $M(x,y,z,t_1+t_2+t_3) \geq M(x,y,z,t_1) * M(x,y,z,t_2) * M(x,y,z,t_3)$,

for all x,y,z in X and t_1,t_2,t_3 in [0,1].It is easily seen that M is non -negative.

Definition(2.4)[3]:A sequence$\{x_n\}$in a fuzzy 2-metric space(X,M,*)is said to be **convergent** with lim x in X if

$$\lim_{n\to\infty} M(x_n,x,z,t)=1, \text{for all } x,z \text{ in} X \text{and } t>0.$$

Definition(2.5)[3]:A sequence $\{x_n\}$ in a fuzzy 2-metric space (X,M,∗) is said to be a **Cauchy sequence** if

$$\lim_{m,n\to\infty} M(x_m,x_n,z,t)=1, \text{for all } x,z \text{ in} X \text{ and } t>0.$$

Definition(2.5)[3]:A fuzzy 2-metric space (X,M,∗)is said to be **complete** fuzzy 2-**metric space** if every Cauchy sequence in X is convergent.

DEFINITION 2.6([1]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,*) are said to be compatible if,

$\lim_{n\to\infty} M(ASx_n, SAx_n,z, t) = 1$ whenever $\{x_n\}$ is a sequence such that,

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.7([4]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,*) are said to be weakly compatible if, they commute at coincidence points.That is,Ax=

S*x* implies that AS*x*=SA*x* for all *x* in X.

DEFINITION 2.8([2]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,*) are said to be subcompatible if, $\lim_{n\to\infty} M(ASx_n, SAx_n,z,t) = 1$ whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

DEFINITION 2.9([7]) Two self-mappings A and S of a fuzzy 2-metric space (X,M,*) are said to be reciprocally continuous if,

$\lim_{n\to\infty} M(ASx_n, Ax, z, t) = 1$

and $\lim_{n\to\infty} M(SAx_n, Sx, z, t) = 1$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

It is noted that if A and S are both continuous, they are obviously reciprocally continuous but the converse need not be true. For this, we have the following example:

EXAMPLE 1.1([7]) Consider two mappings A and S defined on X = [2, 20] with the usual Euclidean metric d, defined by the rule $A_2 = 2, Ax = 3$ if $x > 2$ and $S_2 = 2, Sx = 6$ if $x > 2$. Then, A and S are reciprocally continuous but are not continuous.

DEFINITION 2.10([6]) Two self-mappings A and S of a fuzzy 2-metric space $(X, M, *)$ are said to be compability of type(A) if,

$\lim_{n\to\infty} M(ASx_n, SSx_n, z, t) = 1$

and $\lim_{n\to\infty} M(SAx_n, AAx_n, z, t) = 1$

whenever $\{x_n\}$ is a sequence such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

3. MATERIAL AND METHOD

Theorem(3.1):Let (X,M,∗) a fuzzy 2-metric space and let T,S,A,B be continuous self mappings of X , then $\{Tx_n\},\{Sx_n\},\{Ax_n\}$ and $\{Bx_n\}$ converges to p. If T,S,A and B satisfying the following condition

$M(Tx,Sy,z,t) \le aM(Sy,Ax,z,t)$

$+bM(x.Tx,z,t)+cM(x,Ty,z,t)$

$+dM(y,By,z,t)+eM(Sx,By,z,t)$

s.t. x,y,z in X and $t \in [0,1]$ when $a+b+2c+d+e<1$ then T,S,A and B has a common fixed point p.

4. RESULT AND DISCUSSION

Proof of theorem: Consider $Q_{n-1} = M(x_{n+1},x_{n+2},z,t)$ if $x=x_n$, $y=x_{n+1}$ then after substituting the value we have

$Q_{n+1} \le aQ_n + bQ_n + c(Q_n + Q_{n+1}) + dQ_{n+1} + eQ_{n+1}$

$\Rightarrow (1-a-c-d-e)Q_{n+1} \le (b+c)\, Q_n$

$$\Rightarrow Q_{n+1} \le \frac{b+c}{1-a-c-d-e}\, Q_n$$

$$\Rightarrow Q_{n+1} \le k\, Q_n \text{ when } k = \frac{b+c}{1-a-c-d-e} < 1,$$

Therefore $a+b+2c+d+e<1$

$\Rightarrow Q_n \le k\, Q_{n-1} \Rightarrow Q_{n+1} \le k^2\, Q_{n-1}$

$\Rightarrow Q_n \le k^n\, Q_0$

But $Q_0 = M(x_0,x_1,z,t)$ is a Cauchy fuzzy 2-metric space .So Q_n be Cauchy fuzzy 2-metric space .Also we can easily prove that convergent.

Since $M(Tx_n,p,z,t) \le M(Tx_n,p,z,t/2) * M(Tx_n,p,z,t/4) * M(Tx_n,p,z,t/8) * \ldots\ldots$

$M(Tx_n,p,z,t/2^n)$

$\Rightarrow M(Tx_n,p,z,t) \to 1$

Therefore it converge at the point p . Similaly S,A and B converge to same point p. Therefore we can easily show that point p is unique.so this is a type(A),Since

$M(TSx,SSy,z,t) \le aM(SSy,TAx,z,t)$

$+bM(Tx.TSx,z,t)+cM(Tx,TSy,z,t)$

$+dM(y,By,z,t)+eM(SSx,TBy,z,t)$

Let $x=x_n$, $y=x_{n+1}$ then we have

$M(TSx_n,SSx_{n+1},z,t)$

$\le aM(SSx_{n+1},TAx_n,z,t)+$

$bM(Tx_n.TSx_n,z,t)+cM(Tx_n,TSx_{n+1},z,t)$

$+dM(x_{n+1},B\,x_{n+1},z,t)$

$+eM(SSx_n,TB\,x_{n+1},z,t)$

for x,y,z in X and $t \in [0,1]$

Consider $Q_{n-1} = M(x_{n+1},x_{n+2},z,t)$

Then we have

$(1-a-c-e)Q_{n+1} \le (b+c+d)\, Q_n$

$$\Rightarrow Q_{n+1} \le \frac{b+c+d}{1-a-c-e}\, Q_n$$

$$\Rightarrow Q_{n+1} \le k\, Q_n \text{ when } k = \frac{b+c+d}{1-a-c-e} < 1,$$

therefore $a+b+2c+d+e<1$

$\Rightarrow Q_n \leq k\, Q_{n-1}$

$\Rightarrow Q_{n+1} \leq k^2\, Q_{n-1}$

$\Rightarrow Q_n \leq k^n\, Q_0$

whenever $\{x_n\}$ is a sequence $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = p$, for some p in X.

So we have prove that this is a compability of type(A).

5. CONCLUSION

As Q_n or $\{Tx_n\}_{n=1}^{\infty}$ is a Cauchy sequence, so $Tx_n=p$.

$\Rightarrow M(Tx_n,p,z,t) \to 1$

And $M(TSx_n,SSx_n,z,t) \to 1$

Therefore T,S,A and B has a common fixed point p.as well as T,S,A and B are compability of type(A)

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