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Weak Contraction in Fuzzy b-Metric Space

Gauri Shanker Sao

Dept. of Mathematics, Govt. ERR Pg Sc. College Bilaspur (C.G.)

Aradhana Sharma

Dept. of Mathematics, Govt. Girls Pg College, Bilaspur

Abstract: *The aim of present paper is to obtain a fixed point by weak contraction in fuzzy b-metric space. My purpose is generalizing the several known results.*

Keywords and Phrases: *fuzzy b-metric spaces, weak contraction, fixed point.*

AMS subject classification (2000): *54H25 , 47H10.*

1. INTRODUCTION

The fundamental work for the fuzzy theory was first given by Zadeh [6] in 1965, who introduced the concept of fuzzy set. Czerwik,S [4-5] developed the b-metric space and later George and Veeramani [1] modified the notion of fuzzy metric spaces by introducing the concept of continuous t – norm.Now define fuzzy b-metric space.

2. PRELIMINARIES

Definition(2.1):A binary operation : $[0,1]x[0,1] \to [0,1]$ is a continuous t-norm if $\{[0,1],*\}$ is an abelian topological monoid with unit 1 such that $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$, $a,b,c,d \in [0,1]$

Definition(2.2):A **fuzzy metric space** is a space X in which for each triple of points x,y,z there exits a real function M(x,y,t) such that :

(i)to each pair of distinct points x,y in there exists a point t in[0,1] then $M(x,y,t) \neq 0$;

(ii) M(x,y,t) = 1, when x,y are equal;

(iii)M(x,y,t)= M(y,x,t) ;

(iv)$M(x,y,t_1+t_2+t_3) \geq M(x,y,t_1) * M(x,y,t_2) * M(x,y,t_3)$,for all x,yin X and t_1,t_2,t_3 in [0,1].It is easily seen that M is non -negative.

Definition(2.3):A sequence$\{x_n\}$in a fuzzy metric space(X,M,∗)is said to be **convergent** with lim x in X if $\lim_{n\to\infty} M(x_n,x,t)=1$,for all z in X and t>0.

Definition(2.4):A sequence $\{x_n\}$ in a fuzzy metric space (X,M,∗) is said to be a **Cauchy sequence** if $\lim_{m,n\to\infty} M(x_m,x_n,t)=1$,for all z in X and t>0.

Definition(2.5):A fuzzy metric space (X,M,∗)is said to be **complete** fuzzy **metric space** if every Cauchy sequence in X is convergent.

Definition(2.6): A fuzzy metric space (X,M,∗)is said to be weak **contraction** fuzzy b-**metric space** if there exit $s \in (0,1)$ and G≥0

$$M(Ax,Ay,t) \geq sM(x,y,t)*GM(y,Ax,t)$$

OR

$$M(Ax,Ay,t) \geq sM(x,y,t)*GM(x,Ay,t)$$

for all $x,y \in X$ and $A:X \to X$.

3.MATERIAL AND METHOD

Theorem(3.1):Let (X,M,∗) be a fuzzy b-metric space satisfying the following Kannan type conditions

$$M(Ax,Ay,t) \geq s[M(x,Ax,t)+M(y,Ay,t)]$$

There exit $s \in (0,1)$ for all $x,y \in X$ and $A:X \to X$.Then $x^* \in X$ such that $x_n \to x^*$ and x^* is unique fixed point of A.

3. RESULT AND DISCUSSION

Proof of theorem: Let $x_0 \in X$ and $\{x_n\}_{n=1}^{\infty}$ be a sequence in X defined as $x_n=Ax_{n-1}=A^nx_0$ For n=1,2,3………….

Then $A^0x_0=x_0$,$Ax_0=x_1$

So $M(x_n,x_{n+1},t)=M(Ax_{n-1},Ax_n,t)$

$\geq \mu[M(x_{n-1},x_n,t)$

$+M(x_n,x_{n+1},t)]$

Hence $M(x_n,x_m,t)>sM(x_n,x_{n+1},t)$

$+s^2M(x_{n+1},x_{n+2},t)+s^3M(x_{n+2},x_{n+3},t)+\dots$

$>sk^nM(x_0,x_1,t)+s^2k^{n+1}M(x_0,x_1,t)+\dots\dots\dots$

Therefore $M(x^*,Ax^*,t)\geq s[M(x^*,x_n,t)+M(x_n,Ax^*,t)]$

$\geq sM(x^*,x_n,t)+s\mu[M(x_{n-1},x_n,t)+M(x_n,Ax^*,t)]$

$$\geq \frac{s}{1-s\mu}M(x^*,x_n,t)+\frac{s\mu}{1-s\mu}M(x_n,Ax^*,t)$$

$$\geq \frac{s}{1-s\mu}M(x^*,x_n,t)$$

$$+\frac{s\mu}{1-s\mu}\left(\frac{\mu}{1-\mu}\right)^n M(x_0,x_1,t)$$

4. CONCLUSSION

$Ax^*=\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Ax_{n+1}=x^*$

Therefore $x^*\in X$, $x_n\to x^*$ and x^* is unique fixed point of A.

Remark: If $M(Ax,Ay,t)\geq\mu[M(x,Ax,t)*M(y,Ay,t)]$ is contractive need not weak contractive in fuzzy b-metric space when $\mu s\in[0,\frac{1}{2})$

Since $M(Ax,Ay,t)\geq\mu[M(x,Ax,t)+M(y,Ay,t)]$

$\geq\mu[sM(x,y,t)+sM(y,Ax,t)+M(y,Ay,t)]$

$\geq\mu[sM(x,y,t)+2sM(y,Ax,t)+sM(x,Ay,t)]$

$$\geq\frac{s\mu}{1-s\mu}M(x,y,t)+\frac{2s\mu}{1-s\mu}M(y,Ax,t)$$

Not weak contraction unless the term $\mu s\in[0,\frac{1}{2})$

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